Group #: _____ Members: _____ Rating: _____

1. Fill in the blanks.
   (a) (1 point) A square matrix A is diagonalizable if _____.
2. (4 points) Determine whether the following matrices are diagonalizable (over the real numbers). If so, diagonalize it by finding the appropriate P (invertible) and D (diagonal) matrices so that $A = PDP^{-1}$. If not, explain why not.
   (a) $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.
   (b) $B = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -2 \\ 1 & 3 & 1 \end{bmatrix}$.
3. **Prove** or **disprove** (i.e., give a counterexample) the following statements.
   (a) (2 points) If a $n \times n$ matrix A is diagonalizable, then A is invertible.
   (b) (2 points) If a $n \times n$ matrix A is invertible, then A is diagonalizable.
   (c) (1 point) If a $n \times n$ matrix A does not have n distinct eigenvalues, then A is not diagonalizable.