Group #: ________ Members: ______________________________ Rating: ___________

1. (4 points) Given $A = \begin{bmatrix} -4 & 3 \\ 2 & 1 \end{bmatrix}$.
   (a) (2 points) Find the characteristic polynomial of A and its eigenvalues.
   (b) (2 points) For each eigenvalue, find a **basis** for its associated eigenspace. (Note: Your answer for the basis should be a set of finitely many vectors.)
2. (3 points) In this problem, let's explore what it means for a matrix to have an eigenvalue of 0.
   - A has an eigenvalue of 0 means $Ax = 0x = 0$ has a non-trivial solution.
   - But $Ax = 0$ has a non-trivial solution if and only if A is ____________________.
   - Hence, A is _______________ if and only if A does not have an eigenvalue of 0.

   Please add this fact to the invertible matrix theorem and use it to find an eigenvalue of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}$ **without any calculation**. Justify your answer.
3. (3 points) In this problem, let's examine a class of matrices whose eigenvalues are readily available.
   (a) What are the eigenvalues of a diagonal matrix $D = \begin{bmatrix} d_{11} & 0 & 0 & \dots & 0 \\ 0 & d_{22} & 0 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \dots & d_{nn} \end{bmatrix}$ and why?
   (b) What are the eigenvalues of a triangular matrix (either upper or lower), e.g., an upper triangular matrix $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \dots & u_{1n} \\ 0 & u_{22} & u_{23} & \dots & u_{2n} \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \dots & u_{nn} \end{bmatrix}$ and why?

   Use these facts to construct an invertible 5×5 matrix that has an eigenvalue of algebraic multiplicity two and an eigenvalue of algebraic multiplicity three.