

Group #: \_\_\_\_\_ Members: \_\_\_\_\_ Rating: \_\_\_\_\_

1. (2 points) Let  $b_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$ ,  $b_2 = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$ ,  $c_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ ,  $c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  be such that  $\mathcal{B} = \{b_1, b_2\}$  and  $\mathcal{C} = \{c_1, c_2\}$  are two bases for  $\mathbb{R}^2$ . Given that  $[x]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ , find  $[x]_{\mathcal{C}}$ .
2. (4 points) In  $\mathbb{P}_2$ , find the change-of-coordinates matrix from the basis  $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$  to the standard basis. Then write  $t^2$  as a linear combination of the polynomials in  $\mathcal{B}$ .
3. (4 points) Let  $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\}$  and  $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3\}$  be bases for a vector space  $V$ , and suppose  $\mathbf{f}_1 = 2\mathbf{d}_1 - \mathbf{d}_2 + \mathbf{d}_3$ ,  $\mathbf{f}_2 = 3\mathbf{d}_2 + \mathbf{d}_3$ , and  $\mathbf{f}_3 = -3\mathbf{d}_1 + 2\mathbf{d}_3$ .
  - (a) (2 points) Find the change-of-coordinates matrix from  $\mathcal{F}$  to  $\mathcal{D}$ .
  - (b) (2 points) Find  $[\mathbf{x}]_{\mathcal{D}}$  for  $\mathbf{x} = \mathbf{f}_1 - 2\mathbf{f}_2 + 2\mathbf{f}_3$ .