Group #: ________ Members: ______________________________ Rating: ___________

1. (3 points) Let $A = \begin{bmatrix} 2 & -1 & 1 & -6 & 8 \\ 1 & -2 & -4 & 3 & -2 \\ -7 & 8 & 10 & 3 & -10 \\ 4 & -5 & -7 & 0 & 4 \end{bmatrix}$.

   (a) Find a basis for Col A and rank A.

   (b) Find a basis for Nul A and dim Nul A.

   (c) Verify the rank theorem.

2. (2 points) Find the dimension of the subspace of all vectors in $\mathbb{R}^5$ whose first, second, and forth entries are equal. Make sure you justify your answers.

3. (1 point) The statement "$\mathbb{R}^2$ is a two-dimensional vector subspace of $\mathbb{R}^3$" is false. Correct the statement to make it true.

4. (2 points) Mark each statement **True** or **False**. **Justify** your answer. Be sure to provide a counterexample for a false statement.

   (a) The sum of the dimensions of the row space and the null space of A equals the number of rows in A.

   (b) If B is any echelon form of A, then the pivot columns of B form a basis for the column space of A.

5. (2 points) The rank theorem gives a counting rule that is useful in getting a sense of what the solution to a large system looks like before having to solve it. For example, a scientist has found two **independent solutions** to a homogeneous system of 40 equations in 42 variables. All other solutions can be constructed as a linear combination of these two solutions. Can the scientist be certain that an associated non-homogeneous system has a solution? Why or why not?