Group #: ________ Members: ______________________________ Rating: __________

1. (Definitions) Fill in the blank.

   (a) (1 point) The coordinates of x relative to the basis $\mathcal{B} = \{b_1, b_2, \ldots, b_p\}$ for the vector space V are ____________________.

2. (2 points) Write 329 in Roman Numerals given that some of its basis elements are

I	1
V	5
X	10
L	50
C	100
D	500
M	1000

3. (4 points) Let $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$, $b_3 = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$, and $x = \begin{bmatrix} 8 \\ 2 \\ 3 \end{bmatrix}$.

   (a) Show that $\mathcal{B} = \{b_1, b_2, b_3\}$ is a basis of $\mathbb{R}^3$.

   (b) Find the change-of-coordinate matrix from $\mathcal{B}$ to the standard basis and use it to write the equation that relates $x \in \mathbb{R}^3$ to $[x]_{\mathcal{B}} \in \mathbb{R}^3$.

   (c) Use the equation you obtained in part (b) to find $[x]_{\mathcal{B}}$, for the x given above.

4. (3 points) The first four Hermite polynomials are 1, $2t$, $-2 + 4t^2$, and $-12t + 8t^3$. These polynomials arise naturally in the study of certain important differential equations in mathematical physics.

   (a) Use the coordinate mapping to show that the first four Hermite polynomials forms a basis of $\mathbb{P}_3$.

   (b) Find $[9t^2 + 7t - 1]_{\mathcal{B}}$, where $\mathcal{B}$ contains the first four Hermite polynomials. What does this answer represent?

   (c) Describe what you would have done for part (a) if you weren't allowed to use the coordinate mapping.

5. (Extra credit 3 points) Show that for any $\mathcal{B} = \{b_1, \ldots, b_n\}$, a basis for V, the coordinate mapping $\phi : V \to \mathbb{R}^n$ that takes $x \in V$ to $[x]_{\mathcal{B}} \in \mathbb{R}^n$ is an **isomorphism**, i.e., ϕ is one-to-one, onto, and linear.