Group #: ________ Members: ______________________________ Rating: __________

1. (Definitions) Fill in the blanks.
   (a) (1 point) An indexed set of vectors $\mathcal{B} = \{b_1, b_2, \ldots, b_p\}$ is a basis for a vector space H if ____________________.
2. (1 point) Use your definition in part 1. to construct a non-standard basis for $\mathbb{R}^4$.
3. (5 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix}$.
   (a) (2 points) Find a basis for $\mathrm{N}(A)$. Justify your answer.
   (b) (2 points) Find a basis for $\mathrm{Col}(A)$. Justify your answer.
   (c) (1 point) Use the results from the previous two questions to determine whether A a one-to-one and onto map. Justify your answers.
4. (2 points) Let $p_1(x) = 3, p_2(x) = 2 + x, p_3(x) = -1 + 2x + 4x^2$, and $p_4(x) = -5x^3$. Determine whether $\{p_1, p_2, p_3, p_4\}$ is a basis for $\mathbb{P}_3$ (the set of all polynomials of degree less than or equal to 3). Justify your answer.
5. (1 point) In the vector space of all real-valued functions, find a basis for the subspace spanned by $\{\sin t, \sin 2t, \sin t \cos t\}$.