Group #: ________ Members: ______________________________ Rating: __________

1. (Definitions) Fill in the blanks.
   (a) (1 point) The null space of an $m \times n$ matrix A is ____________________.
   (b) (1 point) The column space of an $m \times n$ matrix A is ____________________.

2. (3 points) Let $A = \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$, $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$.
   (a) Is $u \in \mathrm{N}(A)$? Why or why not? Could u be in $\mathrm{Col}(A)$? Why or why not?
   (b) Is $v \in \mathrm{Col}(A)$? Why or why not? Could v be in $\mathrm{N}(A)$? Why or why not?
   (c) Find a nonzero vector in $\mathrm{N}(A)$ and verify your answer.

3. (2 points) Let A be an $m \times n$ matrix. Using the language of null space and column space to complete the following sentences and justify your answers.
   (a) (Existence) The matrix equation $Ax = b$ has a solution for every $b \in \mathbb{R}^m$ if and only if _____________.
   (b) (Uniqueness) The matrix equation $Ax = b$ has a unique solution for every $b \in \mathbb{R}^m$ if and only if _____________.

4. (3 points) Prove that the column space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^m$. (Hint: #29 in §4.2 exercises.)