Group #: _____ Members: _____ Rating: _____

1. (Definitions) Fill in the blanks.

   (a) (2 points) Let V denote a vector space and H a subset of V. H is a subspace of V if _____.

2. (2 points each) Let V denote a vector space and H a subset of V. Determine if H is a subspace of V in the following items. Be sure to justify your answer.

   (a) $V = \mathbb{R}^2$. $H =$ the union of the first and third quadrants in the xy-plane, i.e.,

   $$H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}.$$

   (b) $V = \mathbb{P}_n$ (the set of all polynomials of degree at most n). $H =$ the set of polynomials of degree at most n such that $p(0) = 0$, i.e.,

   $$H = \{p(t) = a_0 + a_1 t + \ldots + a_n t^n : p(0) = 0 \quad \text{and} \quad a_0, a_1, \ldots, a_n \in \mathbb{R}\}.$$

   (c) $V = \mathbb{P}_2$ (the set of all polynomials of degree at most 2). $H = \{p(t) = a + t^2 : a \in \mathbb{R}\}$.

   (d) $V = \mathbb{R}^3$. $H = \left\{ \begin{bmatrix} 1 \\ 3a - 5b \\ 3b + 2a \end{bmatrix} : a, b \in \mathbb{R} \right\}$.

   (e) $V = \mathbb{R}^4$. $H = \left\{ \begin{bmatrix} 4a + 3b \\ 0 \\ a + 3b + c \\ 3b - 2c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$.