Group #: ________ Members: ______________________________ Rating: ___________

1. (3 points) Prove the following.
   (a) If A is invertible, then A^{-1} is also invertible and $\left(A^{-1}\right)^{-1} = A$.
   (b) If A and B are $n \times n$ invertible matrices, then so is AB, and $(AB)^{-1} = B^{-1}A^{-1}$.
   (c) If A is invertible, then so is A^T and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

2. (2 points) Let $A = \begin{bmatrix} -1 & -7 & -3 \\ 2 & 15 & 6 \\ 1 & 3 & 2 \end{bmatrix}$. Find the second column of A^{-1} without computing the other columns.

3. (3 points) A message has been encoded using the matrix

$$C = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 5 & -1 & -1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix}.$$

   The encoded message is

X	H	L	T	A	B	U	T	F	M	Y	Q
G	D	R	E	S			V	G	E	Z	X

   Your assignment (should you choose to accept it) is to decode it.

4. (2 points) Recall that to **formally** justify a **true** *if-then* statement, you need to cite appropriate results (typically in the form of theorems) and show that the antecedent (what follows the "if") in the True/Flase statement is satisfied. This may be hard to do in some situations. On the other hand, to justify a **false** *if-then* statement, you just need to come up with **one** counterexample (i.e., example that shows the statement is not true). Note, you **can not** justify a true statement by simply showing that a few examples work (Getting heads on ten consecutive coin tosses does not mean that you won't get tails on the 11th toss.). Now, pove or disprove the following statements.
   (a) If A is invertible, then so is $A^3 + I$.
   (b) Let A be symmetric and invertible. Then A^{-1} is also symmetric.