Group #: ________ Members: ______________________________ Rating: __________

1. (2 points) Denote the ith row of the matrix B by r_i. Use the definition of row-column matrix multiplication to compute AB where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ -1 & 0 & 2 & 4 \\ 3 & 1 & 6 & 0 \end{bmatrix}$$

   Write your result in terms of r_i's only.

2. (1 point) Suppose the third column of the matrix B is the sum of the first two columns. Show (by way of a direct computation) that the third column of AB is the sum of its first two columns for any matrix A.

3. (5 points) Prove (i.e., provide a sound argument) or disprove (i.e., provide a counterexample) the following statements for matrices A, B, C, and O (zero matrix) of appropriate sizes. Here, we assume all matrix multiplications are compatible, please do not argue that a statement is false based on sizes.
   (a) $(ABC)^T = C^T A^T B^T$.
   (b) If A is an $n \times n$ matrix, then $\left(A^2\right)^T = \left(A^T\right)^2$.
   (c) $AB = BA$ (commutativity).
   (d) If $AB = AC$, then $B = C$ (cancelation property).
   (e) If $AB = O$, then $A = O$ or $B = O$.

4. (2 points) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Notice that D is a diagonal matrix (i.e., nonzero entries on the diagonal and zero elsewhere). **Write AD and DA in terms of rows/columns of the matrix A.** Other than $B = O$, $B = I$, and $B = A$, when might the statement $AB = BA$ be true?