Predicting CSU Tuition using Linear Least Squares Regression

Introduction

We will be using least square regression to approximate the cost of CSU tuition for undergraduate and graduate. Data are gathered between 2001 and 2016. Using the result, we can predict the cost for the year 2026.

Linear least squares regression is a mathematical procedure for find the best fit curve from sets of bivariate data. The equation for the least-square line is $Y = B_0 + B_1 X$.

To find the calculation for this line, the squares of the residuals (distance between the predicted y-value and the observed y-value) are added. It is known as the least-square because we want to minimize the sum of the squares of the residuals.

Methodology

- Historical CSU tuition data for the full-time undergraduate and graduate students have been collected from years 2001 to 2015:

Year	Full-Time Undergraduate Tuition (USD)	Full-Time Graduate Tuition (USD)
2001	1,428	1,506
2002	1,572	1,734
2003	2,046	2,256
2004	2,334	2,820
2005	2,520	3,102
2006	2,520	3,102
2007	2,772	3,414
2008	3,048	3,756
2009	4,026	4,962
2010	4,230	5,214
2011	5,472	6,738
2012	5,472	6,738
2013	5,526	6,738
2014	5,472	6,738
2015	5,472	6,738

- Let $y_u = b_0 + b_1 x$ and $y_g = c_0 + c_1 x$ be the least-squares lines for the full-time undergraduate tuition data and the full-time graduate tuition data, respectively. Then, we have the following:

$$X = [\, 1 \quad x \,] = \begin{bmatrix} 1 & 2001 \\ 1 & 2002 \\ 1 & 2003 \\ 1 & 2004 \\ 1 & 2005 \\ 1 & 2006 \\ 1 & 2007 \\ 1 & 2008 \\ 1 & 2009 \\ 1 & 2010 \\ 1 & 2011 \\ 1 & 2012 \\ 1 & 2013 \\ 1 & 2014 \\ 1 & 2015 \end{bmatrix}, \quad k = \begin{bmatrix} b_0 & c_0 \\ b_1 & c_1 \end{bmatrix}, \quad Y = [\, y_u \quad y_g \,] = \begin{bmatrix} 1428 & 1506 \\ 1572 & 1734 \\ 2046 & 2256 \\ 2334 & 2820 \\ 2520 & 3102 \\ 2520 & 3102 \\ 2772 & 3414 \\ 3048 & 3756 \\ 4026 & 4962 \\ 4230 & 5214 \\ 5472 & 6738 \\ 5472 & 6738 \\ 5526 & 6738 \\ 5472 & 6738 \\ 5472 & 6738 \end{bmatrix}, \quad \text{and } Xk = Y$$

(Year ↑; Full-Time Undergraduate Tuition ↑; Full-Time Graduate Tuition ↑)

- To find the least-squares solution of $Xk = Y$, we obtain the normal equations:

$$X^T X k = X^T Y \quad \Rightarrow \quad k = (X^T X)^{-1} X^T Y$$

- Using the equations of the least-squares lines determined, we estimate the future CSU tuition:

$$y_u = b_0 + b_1 x \qquad y_g = c_0 + c_1 x$$

Substitute Future Year for x

Calculations

$$X^T X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2001 & 2002 & 2003 & 2004 & 2005 & 2006 & 2007 & 2008 & 2009 & 2010 & 2011 & 2012 & 2013 & 2014 & 2015 \end{bmatrix} \begin{bmatrix} 1 & 2001 \\ 1 & 2002 \\ 1 & 2003 \\ 1 & 2004 \\ 1 & 2005 \\ 1 & 2006 \\ 1 & 2007 \\ 1 & 2008 \\ 1 & 2009 \\ 1 & 2010 \\ 1 & 2011 \\ 1 & 2012 \\ 1 & 2013 \\ 1 & 2014 \\ 1 & 2015 \end{bmatrix} = \begin{bmatrix} 15 & 30120 \\ 30120 & 60481240 \end{bmatrix}$$

$$X^T y_u = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2001 & 2002 & 2003 & 2004 & 2005 & 2006 & 2007 & 2008 & 2009 & 2010 & 2011 & 2012 & 2013 & 2014 & 2015 \end{bmatrix} \begin{bmatrix} 1428 \\ 1572 \\ 2046 \\ 2334 \\ 2520 \\ 2520 \\ 2772 \\ 3048 \\ 4026 \\ 4230 \\ 5472 \\ 5472 \\ 5526 \\ 5472 \\ 5472 \end{bmatrix} = \begin{bmatrix} 53910 \\ 108346470 \end{bmatrix}$$

$$X^T y_g = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2001 & 2002 & 2003 & 2004 & 2005 & 2006 & 2007 & 2008 & 2009 & 2010 & 2011 & 2012 & 2013 & 2014 & 2015 \end{bmatrix} \begin{bmatrix} 1506 \\ 1734 \\ 2256 \\ 2820 \\ 3102 \\ 3102 \\ 3414 \\ 3756 \\ 4962 \\ 5214 \\ 6738 \\ 6738 \\ 6738 \\ 6738 \\ 6738 \end{bmatrix} = \begin{bmatrix} 65556 \\ 131757858 \end{bmatrix}$$

$$\begin{bmatrix} 15 & 30120 \\ 30120 & 60481240 \end{bmatrix}\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} 53910 \\ 108346470 \end{bmatrix} \Rightarrow \begin{bmatrix} 15 & 30120 & 53910 \\ 30120 & 60481240 & 108346470 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & -679054 \\ 0 & 1 & 339.96 \end{bmatrix} \quad \begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} -679054 \\ 339.96 \end{bmatrix}$$

$$\begin{bmatrix} 15 & 30120 \\ 30120 & 60481240 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 65556 \\ 131757858 \end{bmatrix} \Rightarrow \begin{bmatrix} 15 & 30120 & 65556 \\ 30120 & 60481240 & 131757858 \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & -866313 \\ 0 & 1 & 433.61 \end{bmatrix} \quad \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} -866313 \\ 433.61 \end{bmatrix}$$

Yearly Full Time Tuition - Undergraduate

$y = 339.96x - 679054$
$R^2 = 0.9387$

Yearly Full Time Tuition - Graduate

$y = 433.61x - 866313$
$R^2 = 0.9463$

Results

$$y_u = 339.96x - 679054$$

$$y_g = 433.61x - 866313$$

- **Tuition Forecast for Year 2020 (Five Years from Now):**

  Full-Time Undergraduate: $y_u = 339.96 * 2020 - 679054 = \$7,665$
  Full-Time Graduate: $y_g = 433.61 * 2020 - 866313 = \$9,579$

- **Tuition Forecast for Year 2025 (Ten Years from Now):**

  Full-Time Undergraduate: $y_u = 339.96 * 2025 - 679054 = \$9,365$
  Full-Time Graduate: $y_g = 433.61 * 2025 - 866313 = \$11,747$

- **Tuition Forecast for Year 2030 (Fifteen Years from Now):**

  Full-Time Undergraduate: $y_u = 339.96 * 2030 - 679054 = \$11,065$
  Full-Time Graduate: $y_g = 433.61 * 2030 - 866313 = \$13,915$

- **Tuition Forecast for Year 2035 (Twenty Years from Now):**

  Full-Time Undergraduate: $y_u = 339.96 * 2035 - 679054 = \$12,765$
  Full-Time Graduate: $y_g = 433.61 * 2035 - 866313 = \$16,083$

According to the linear regression model, the CSU tuition will more than double after twenty years.

Conclusion

The least-squares lines that best fit the 2001 – 2015 CSU tuition data for the full-time undergraduate and graduate students have been found. Using the equations of the least-squares lines, the future CSU tuition has been estimated. From the linear regression model, it can be predicted that the CSU tuition will more than double after twenty years.

Acknowledgement

Lay, David C. *Linear Algebra and its Applications* 4th ed. Page 368. Boston: Pearson Education, Inc., 2012 Print

CSU Budget Office, "Fee Rates." *Fee Rates.* 5 Mar. 2015. Web. 3 May 2015. <http://www.calstate.edu/budget/student-fees/fee-rates/>.

CSU Budget Office, "Financial Aid and Tuition Rates." *Historical State University Fee Rates.* 25 Sept. 2012. Web. 3 May 2015. <http://www.calstate.edu/budget/fybudget/2012-2013/documentation/13-historical-suf-rates.shtml/>.

Elizabeth Villa, Mitsuyo Tsuda, Fred Rodriguez, Khai Nguyen

California State University, Long Beach