Linear Algebra and its Applications in Chess

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Introduction

Chess is a game that has been played throughout the centuries, with each piece having a set number of possible moves based on unique movement patterns. The objective of chess is to deprive the opponent's king of all possible safe moves. Given the set movement pattern of pieces, it is possible to represent these movements as vectors in a vector matrix. Doing so can allow players to make more informed movement decisions based on the vectors in this matrix. It is our objective to determine how the information from this matrix can be used to optimize strategic movement and to help secure a win in endgame scenarios.

Methods

Chess Vector Matrix:

The Vector Matrix is determined by the 8-by-8 game board and by the 32 pieces that the game is played with, giving us a 8 x 8 x 32 vector matrix.

$$\begin{matrix} 0 & (0,n) & 0 \\ (-n,0) & R & (n,0) \\ 0 & (0,-n) & 0 \end{matrix}$$

Piece Movement:

There are 32 pieces in a game of chess, however there are only 6 distinguishable pieces , of which 7 unique vector matrices can be created. The pawn, the rook, the knight, the right bishop, and the left bishop, the queen, and the king, all have distinguished vector matrices. Depending on the piece movement each vector matrix is different. For example, the Rook has a movement that can be represented by the vector $C_n(1,0)$ and $C_n(0,1)$, $(-7 \le C_n \le -1) \cup (1 \le C_n \le 7)$ we consider the initial position of a piece, the origin.

Restricted / Impossible Moves:

Having one piece on a board allows easy movement. However, once we add another piece movement can be restricted based on the rules of the game. This leads to the vector matrix being changed.

Restricted moves: We can represent this by restricting the weight on each direction vector. Each restriction is dependent on the origin and the other pieces on the board.

Impossible: Impossible moves can be represented as an *i*. In other words, there are no linear combinations that can get you to that particular space in one turn.

An example of an endgame solution Matrix:

			K				
		-1	-1	-1			
		-5	0	-5			R

We can compare the endgame scenario to the homogeneous system Ax=0 where A is the 8 x 8 chessboard and x is the column vector with n entries that represent all movements that lead to this end-game solution matrix.

Check/Illegal Move:

R	(1,0)	(2,0)	(3,0)	(4,0)	(5,0)	(6,0)	(7,0)
(0,-1)	i	i	i	i	i	i	i
(0,-2)	i	i	i	i	i	i	i
(0,-3)	i	i	i	i	i	i	i
(0,-4)	i	i	i	i	i	i	i
(0,-5)	i	i	i	i	i	i	i
(0,-6)	i	i	i	i	i	i	i
(0,-7)	i	i	i	i	i	i	i

The term "check" pertains to the King when it is at risk of dying if it does not move. When the King has been checked, the spaces in which it will remain checked will be represented as a -5. This value is arbitrary. A -5 is representative of a type of restricted/ impossible move however, it is a more particular restricted move since doing nothing or moving onto a -5 will end the game, such a move is not allowed.

Illegal move: This is another specific type of restriction pertaining to the King. However this is more particular - when a King moves into a space that will cause it to be checked, such a move is also not allowed.

Why/How This Works

Up to this point, this class has concentrated on matrices that cover two dimensions, meaning that the matrices have two parameters. However, in order to understand the mechanics of chess in terms of matrix algebra, it is important to visualize this as a three dimensional matrix (a matrix with parameters). This would initially be represented by an 8 x 8 x 32 matrix - the 8 x 8 being the spaces on the chess board and the 32 being the pieces on the board. In a sense, each piece has its own 8 x 8 matrix. However, since each of these matrices interact with each other, it is important to represent this as a three dimensional matrix. It is also important to note that the z-dimension (the 32) is dynamic in that the dimension will change as pieces are lost in the game. So in reality this is an 8 x 8 x $(3 \le n \le 32)$ matrix.

Visualizing it as such actually helps optimize piece movement and loss prevention. By looking at this matrix from the top down (along the z-axis) one can essentially see the chess board with all its pieces and their vectors. When choosing to move a piece, you can compare its movement vectors with those of other pieces to determine whether or not the move is permitted, an opponent's piece is captured, or one's own piece is placed in danger in being captured. As such, visualizing it this way actually helps the player predict countermoves and even future moves.

Conclusions

The use of linear algebra and vector matrices helps with the visualization of movements in chess. The use of the 8 x 8 x 32 matrix helps players optimize their piece movement for capture and preservation, as well as predicting one to two moves ahead. However, this math is of limited use when it comes to securing an endgame strategy. While it does help with the visualization of possible moves, it does not lay out a clear path to victory. Defining a formula that creates the end game solution Ax=0 **has** remained elusive for several years.

Acknowledgements

1) Lay, David C. Linear Algebra and Its Applications. 4th Ed. Pearson Education, 2012
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3) Stiller, Lewis. "Multilinear Algebra and Chess Endgames". Games of No Chances, Vol 29. MSRI Publications, 1996. <http://library.msri.org/books/Book29/files/stiller.pdf>