GAME THEORY

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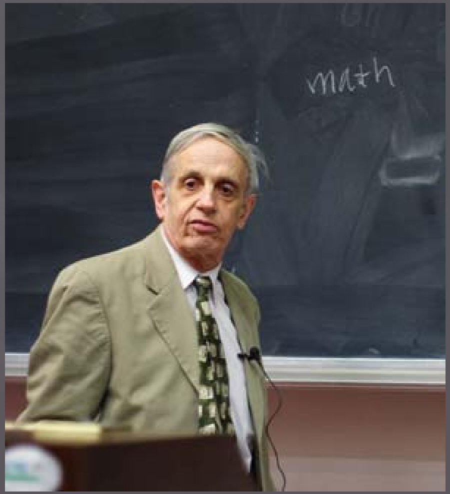
WHAT IS GAME THEORY?

- Game theory uses mathematical models to represent possible outcomes based on the decisions made by intelligent and rational players .
- Player's decision will result in different payoffs or penalties depending on the decisions made by the opposing player.
- Each player will strategically make decisions in order to maximize their gains and minimize their losses.
- In this poster, we will try to illustrate how linear algebra applies to the formulation of mixed strategies within game theory.

HISTORY

- Game Theory was first introduced by John Von Neumann in 1944 and it focused primarily on two-person zero-sum cooperative games.
- In 1950, John Nash further developed the Nash Equilibrium to make Game Theory applicable to, both, non-cooperative and cooperative games.

John V. Neumann



John Nash

ZERO-SUM GAME

The Coin Toss

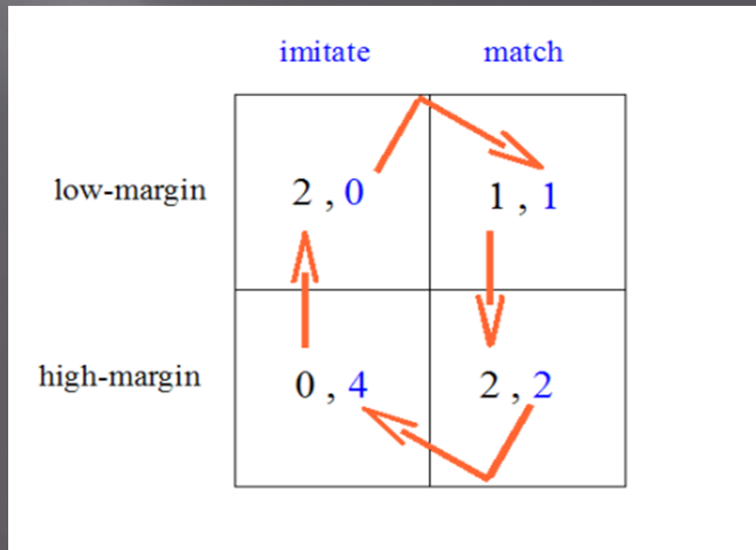
P1/P2	Heads	Tails
Heads	1, -1	0, 0
Tails	0, 0	-1, 1

The "Zero-Sum Game" occurs when the results of a game will add up to zero, no matter what the combination of choices is. The Coin Toss example illustrates this concept perfectly. If each player gets the same result as the other, one will win and the other will lose in equal amounts, and if they each receive opposite results, neither will win.

NASH EQUILIBRIUM

- The refined concept of Nash Equilibrium describes situations in which each player chooses the best decision taking into account its opponent's chosen strategy.
- Nash Equilibrium proves to be most useful in situations where a matrix can no longer be reduced down.
- Each player will find their best respond to each of the opponent's choices.

	imitate	match
low-margin	2 , 0	1 , 1
high-margin	0 , 4	2 , 2



NASH EQUILIBRIUM

P1/P2	Left	Right
Up	3, 3	, 2
Down	2,	, 1

For Player 1

P1/P2	Left	Right
Up	*3, 3	0, 2
Down	2, 0	*1, 1

For Player 2

P1/P2	Left	Right
Up	*3, 3*	0, 2
Down	2, 0	*1, 1*

Therefore, the Nash Equilibrium results are Up, Left and Down, Right. In these Positions, neither player is better off by switching.

METHODS

- We can find the different payoffs for each player by creating an *mxn* matrix A. The columns of A will be the possible moves for X and the rows will be the moves for Y.
- If the entry a_{ij} is positive, it will represent a payment to X. If negative, a payment to Y.
- X can choose any strategy $x = (x_1,\ldots,x_n)$. These give the frequencies for the columns in the matrix and they should add up to 1. Y can do the same thing by choosing a strategy from $y=(y_1,\ldots,y_m)$ and those frequencies should also add up to 1.

•The combination of column j for X and i for row Y will give the probability $x_j y_i$. This gives us the expected payoff of $a_{ij} x_j y_i$ and the total expected payoff as $\sum a_{ij} x_j y_i$.

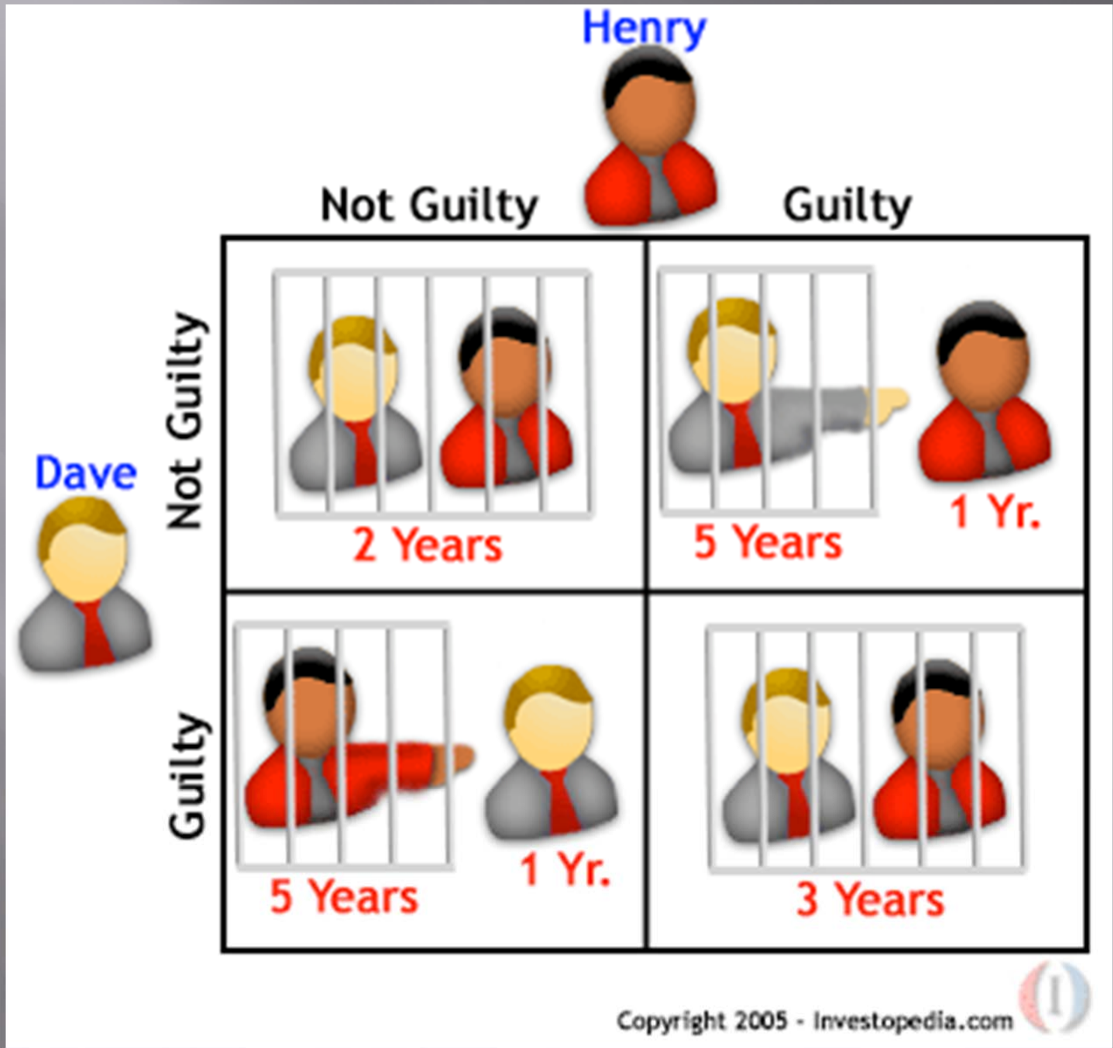
This give us the result:

$$yAx = \begin{bmatrix} y_1 & \cdots & y_m \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \begin{aligned} &= a_{11} x_1 y_1 + \cdots + a_{mn} x_n y_m \\ &= \textbf{average payoff}. \end{aligned}$$

Prisoner's Dilemma

The "Prisoner's Dilemma" is the case where two criminals are arrested for a crime. The investigator does not have enough evidence to convict each criminal for a more serious crime without the cooperation of the other. Each prisoner is offered an opportunity to confess. If none of them confess, they will only be jailed for 2 years. If only one confesses, he will be jailed for only 1 year while the other is jailed for 5. If they both confess, they will each get 3 years in prison. What would you do?

PRISONER'S DILEMMA



SUMMARY/CONCLUSION

- Game theory studies crucial interaction between multiple players. Linear algebra was applied to Game theory to model situations in which players make rational decisions resulting in payoffs or penalties for each participant at each move. A rational player will make decisions that will maximize its expected utility. Linear algebra comes into play where matrices are used to formulate expected pay-offs and total expected pay-offs.

Summary/Conclusion…

- The prisoner's Dilemma was exemplified using a 2x2 matrix where each player has the option to confess or to not confess. The Prisoner's Dilemma becomes a non-zero sum game when the rational prisoners act in their best interest to minimize jail time.
- Game theorists study the concept of Nash equilibrium to examine what the outcome would be had the players taken into account the opponent's decision. A 2x2 matrix was used to illustrate strategies formulated in a 2-player-game.

ACKNOWLEDGEMENTS

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