One Face, Many Vectors

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Introduction

- Purpose: to develop a scheme for facial recognition and get important info on a face image
- Goal: to encode info from a group of faces to distinguish them from one another
- Result: transforms face images into eigenfaces => the training set of face images
- Overview: eigenfaces encode and decode face images that gives info on the content of face features

The Math behind the Eigenface

- eigenvectors of the covariance matrix form main components of the face distributions
  - basically, it characterizes the variation in face images
- eigenvectors are created by each image location
- eigenvectors together form the eigenface
- face images in the training set are represented in a linear combination of the eigenfaces
- number of possible eigenfaces equals number of face images in training set
  - use fewer eigenfaces for computation efficiency => want to compute cheaply and quickly!

Checking for Eigenfaces

- eigenfaces span an *m*-dimensional subspace of the image space from the *m* largest eigenvalues => creates the face space
- For new faces/images:
  - calculate the weights of the new image and the amount of eigenfaces you already have and project the new image onto each eigenface
  - determine if the image is a face and check to see if it is the face space
    - if found to be a face, you can classify the weight pattern to see if it matches that of a face you already have

Method

$$y_1 = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nn} \end{bmatrix} a\ face\ in\ n \times n\ space$$

- This matrix *y* represents a face of *nxn* pixels

$$y_1 \xrightarrow{concatenation} > f_1 = \begin{bmatrix} y_{11} \\ \vdots \\ y_{n1} \\ \vdots \\ y_{nn} \end{bmatrix} \in Rnn$$

- The concatenation process takes the full face matrix and condenses it into a single vector in $\mathbb{R}^{nn}$

Method

$$x = [f_1 \quad f_2 \quad \cdots \quad f_p]$$

- *x* is the “Training Set,” which consists of *p* face matrices *f*.

$$z = x - m$$

$$m = mean(x)$$

- This represents removing the mean face from the training set, which sets them to a common origin.

Method

$$x^{\mathrm{T}}x = [f_1 \quad \cdots \quad f_p]\begin{bmatrix} f_1 \\ \vdots \\ f_p \end{bmatrix}$$

$$x^{\mathrm{T}}x = \begin{bmatrix} f_1 f_1 & \cdots & f_1 f_p \\ \vdots & \ddots & \vdots \\ f_p f_1 & \cdots & f_p f_p \end{bmatrix}$$

- By taking $x^{\mathrm{T}}x$, a matrix is formed that contains the inner products for all *f*.
- The resulting matrix shows a difference of each matrix from the others. If two matrices are the same, their entry will be 0.

Method

$$eig(x^{\mathrm{T}}x) = \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$$

$$\Downarrow$$

$$v_1 \quad v_2 \quad \ldots \quad vn$$

- The eigenvalues and eigenvectors of the $x^{\mathrm{T}}x$ matrix are found and ordered. The amount of vectors and values are usually truncated to a reasonable number

Method

$$span\{v_1 \quad \dots \quad v_n\} = face\ space$$

- The face space is the space that is spanned by the eigenvectors derived from the face set, or eigenfaces, since these vectors represent faces.

Method

$$y_{new} = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nn} \end{bmatrix}$$

$$y_{new} -> fne_w = \begin{bmatrix} y_{11} \\ \vdots \\ y_{n1} \\ \vdots \\ y_{nn} \end{bmatrix}$$

- Using these ideas, any new face could be interpreted in a similar way to the previous faces to find if it matches a known face.

Method

$$f_{new} \in face\ space$$

$$proj_{face\ space fnew} = [p_{new}]$$

$$[p_{new}] - [p_i] = [value]$$

$$where\ [pi] = projfspacefi$$

- By projecting the new face into face space, the difference in the projections can be calculated and compared with known faces. The closest match (smallest *value*) is probably the matching face.

Results

- Each person has a collection of pictures which has an optimal basis for the eigenvectors of the face.
- A photo database of people is developed and if a picture is taken it's eigenvectors can be compared to the bases in the database and the deviation can be used to establish a close or exact match of subject.

Summary/Conclusion

- Face recognition is based on a set image features that approximate known face images the best
  - Eigenfaces present the best solution to determining face recognition
  - Reasons why: it's fast, simple, and works well
- Eigenfaces can be used also for face detection and image compression (to save space)

Acknowledgements

- Chang, Jen-Mei. "Eigenfaces." Faculty Office 3, CSU Long Beach. 26 Apr. 2012
- Kun, Jeremy. "Eigenfaces, for Facial Recognition." Math Programming. 27 July, 2011. 26 Apr. 2012. http://jeremykun.wordpress.com/2011/07/27/eigenfaces/
- Turk, Mathew A., Pentland, Alex P. "Face Recognition Using Eigenfaces" http://www.cs.tau.ac.il/~shekler/Seminar2007a/PCA%20and%20Eigenfaces/eigenfaces_cvpr.pdf











Mean Image

Deviations From Mean Image