

INTRO

Definitions:

Chaos - is turmoil, turbulence, primordial abyss, and undesired randomness, but scientists will tell you that chaos is something extremely sensitive to initial conditions. Chaos also refers to the question of whether or not it is possible to make good long-term predictions about how a system will act.

Gamma (Γ) - Is the matrix used to shear the image.

Modulo (Mod) - The modulo operation finds the remainder of division of one number by another. This is used for keeping the new sheared/distorted pixels back within the boundaries.

Period - number of iterations needed to get back to original

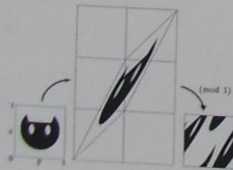
Was discovered by Vladimir Arnold, who demonstrated the effects in the 1960s using an image of a cat, hence the name.

Vladimir Arnold (1937-2010): Was a Soviet and Russian mathematician.

METHODS

$$\Gamma \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \pmod 1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \pmod 1$$

$$= \begin{pmatrix} x + y \\ x + 2y \end{pmatrix} \pmod 1$$



CHAOS / ARNOLD'S

CAT MAP

BY KANE KELLER, ARMÁNDO ZAVALA,
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Dimensions of nxn Matrix

300 x 300
183 x 183
124 x 124
100 x 100
257 x 257
157 x 157
147 x 147
150 x 150
101 x 101

Number of iterations to return to original image

300
60
15
150
258
157
56
300
25

EXAMPLE # 1

This is a transformation of a 124 x 124 pixels picture that uses 15 alliterations to distort the image until it restores it to its original condition



EXAMPLE # 2

A transformation picture of 101 x 101 pixels it takes 25 iterations



RESULTS

Calculation for a single pixel of a 124 x 124 size picture.

$$\Gamma \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix} \pmod{124} = \begin{pmatrix} 27 + 8 \\ 27 + (2 \cdot 8) \end{pmatrix} \pmod{124} = \begin{pmatrix} 35 \\ 45 \end{pmatrix}$$

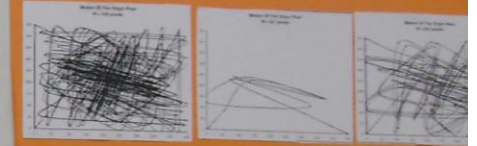
Iteration # 1

$$\begin{pmatrix} 81 \\ 2 \end{pmatrix} \rightarrow \begin{pmatrix} 83 \\ 85 \end{pmatrix} \rightarrow \begin{pmatrix} 44 \\ 5 \end{pmatrix} \rightarrow \begin{pmatrix} 49 \\ 54 \end{pmatrix} \rightarrow \begin{pmatrix} 103 \\ 38 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 12 \\ 45 \end{pmatrix} \rightarrow \begin{pmatrix} 17 \\ 102 \end{pmatrix} \rightarrow \begin{pmatrix} 35 \\ 13 \end{pmatrix} \rightarrow \begin{pmatrix} 38 \\ 41 \end{pmatrix} \rightarrow \begin{pmatrix} 75 \\ 129 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 92 \\ 77 \end{pmatrix} \rightarrow \begin{pmatrix} 108 \\ 63 \end{pmatrix} \rightarrow \begin{pmatrix} 45 \\ 106 \end{pmatrix} \rightarrow \begin{pmatrix} 27 \\ 9 \end{pmatrix}$$

Back to the start after 15 iterations.



CONCLUSION

Arnold's Cat Map on its' own has no real practical use. It does however allow us to understand chaos more in our universe and even though it seems random, there is order/pattern. Chaos allows us to approach things such as the beating of the heart, Red Spots of Jupiter, turbulence of fluids, things of this nature that seem chaotic to bring us closer to understanding our universe.

References:

<http://www.khanacademy.org/a/chaos-theory>

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