Applications of Linear Algebra: Genetics

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Introduction

Genetics is a branch of biology that deals with the mechanisms of heredity transmission and the variation of inherited characteristics among a single organism, species or groups. Throughout time, people have always had an interest in genetics and its effect on their physical traits and inheritable characteristics. As organisms reproduce, new organisms inherit some of their dominant traits and lose recessive ones. In our project, we will be using linear algebra to discuss the effects of reproduction on horse color.

Horses start with two colors: black and bay. The black horse has the recessive genes while the bay horse has either dominant or hybrid genes. Within the first round of combinations black color disappears and all the horses will have the bay color. Bay is a horse with a black mane and tail and a brown coat. Using matrices we will analyze those changes in coat color throughout reproduction.

Methods

Let E = Dominant Allele
Let e = Recessive Allele
When crossing animals, the offspring gets one gene from each parent.
EE genotype horses are the dominant color: Bay
Ee genotype horses are a hybrid horse with the dominant color: Bay
ee genotype horses are the recessive color: Black

Case 1: Consider crossing an EE horse with another EE horse. The probability of getting EE, Ee, and ee will result in a <1,0,0> vector.
Case 2: Consider crossing an EE horse with an Ee bay horse. The probability of getting EE, Ee, and ee will result in a <½,½,0> vector.
Case 3: Consider crossing an EE bay horse with an ee black horse. The probability of getting EE, Ee, and ee will result in a <0,1,0> vector.

Key Terms

Allele: One member of a pair of genes occupying a specific spot on a chromosome that controls the same trait.
Phenotype: The expression of a particular trait, for example, skin color, height, behavior, etc., according to the individual’s genetic makeup and environment.
Genotype: A set of alleles that determines the expression of a particular characteristic or trait.
Recessive Allele: A recessive Allele results in expression of the recessive characteristic if there is no dominant allele present. Trait in organism won’t appear in the presence of a dominant allele.
Dominant Allele: The dominance of an allele over a recessive Allele.
Hybrid Allele: An offspring resulting from the cross between parents of different alleles, for example EE and ee, Ee and ee, or EE and Ee.

Summary

For our poster project, we used linear algebra to predict the different outcomes of genetic traits among horse hair color. In each of our three cases we “bred” two horses:

Case 1: Two horses with dominant alleles.

Case 2: One horse with dominant alleles and one with hybrid alleles.

Case 3: One horse with dominant alleles and one with recessive alleles.

In order to determine the different options for hair color in these horses’ offspring, we fit the different allele types into matrices.

Conclusions

We found over time, the dominant horse will outbreed the recessive horse because of the characteristics of its dominant alleles. According to our example, once the horses have bred one time, our data shows that in their offspring, the recessive gene is entirely gone. Furthermore, every time these same horses breed, the dominant gene occurs more often and the hybrid gene less among their offspring

After years of continuous breeding, the dominant gene will be present as the strongest element among these horses.

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Acknowledgement

"Application to Genetics." Université D'Ottawa. Web. 20 Apr. 2012. <http://aix1.uottawa.ca/~jkhoury/genetics.htm>.

"Biology Forum." Life Science Reference. Web. 20 Apr. 2012. <http://www.biology-online.org/>.

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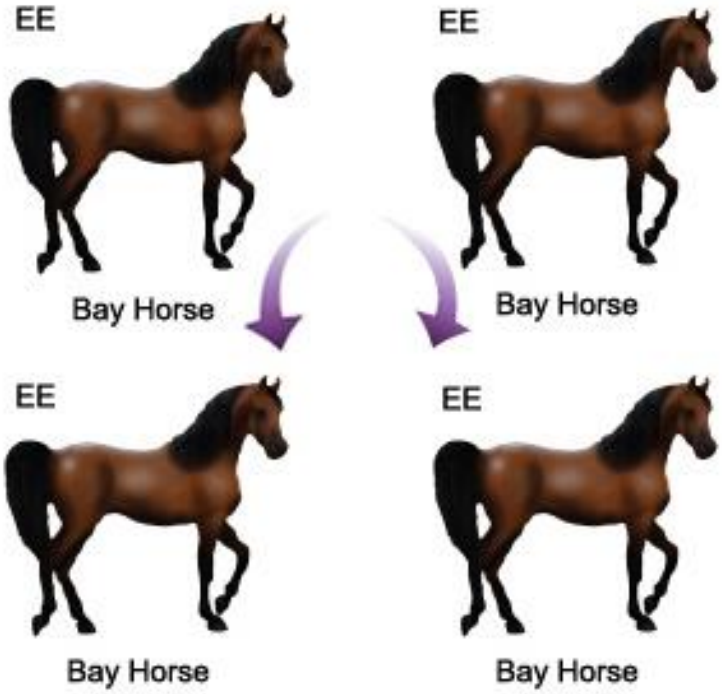
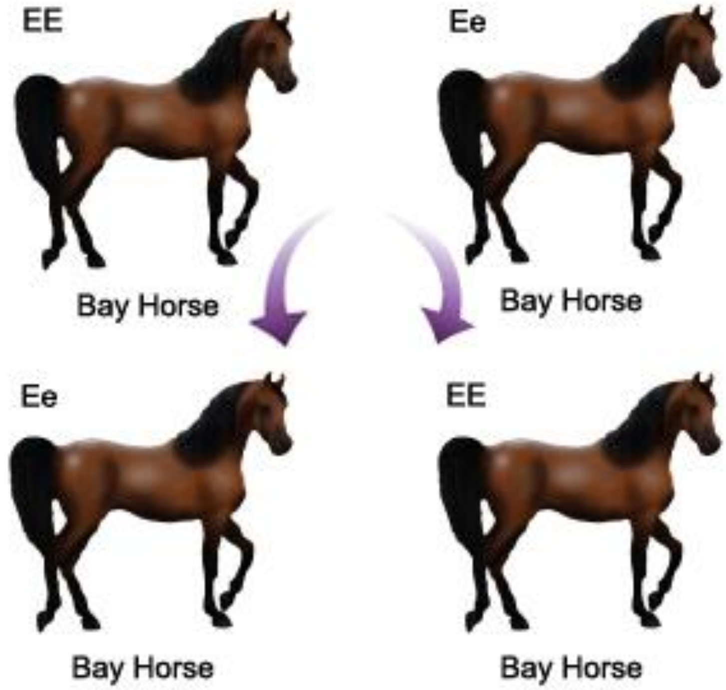
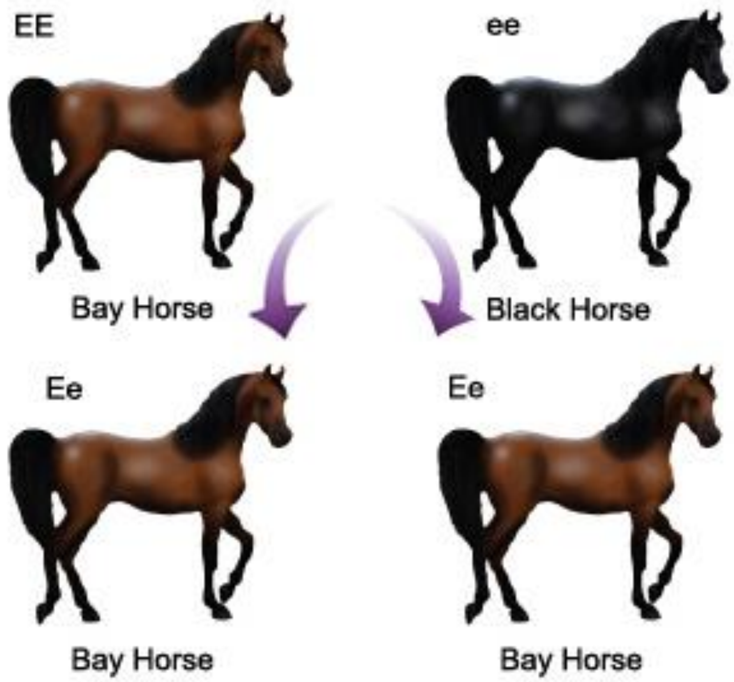
Case 1



Case 2



Case 3



There are 3 possibilities: EE, Ee, and ee which are all equally as likely to occur hence the initial matrix x_0 where the entries represent EE, Ee and ee genotypes and x_n (n is the number of years).

$$x_0 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Crossings			Genotype
Case 1: EE with EE	Case 2: EE with Ee	Case 3: EE with ee	
1	$1/2$	0	EE
0	$1/2$	1	Ee
0	0	0	ee

The resulting matrix is:

$$E = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

After one year, the genotype distribution can be found by solving $Ex_{0} = x_1$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

To find the distribution for the next year, use the current distribution and multiply by the E matrix. Continue this process every year for as many years as desired.

Generally the distribution of the genotype is found by

$$x_n = Ex_{n-1}$$

Over time, the distribution is:

$$x_2 = \begin{bmatrix} \frac{3}{4} \\ \frac{1}{4} \\ 0 \end{bmatrix}, x_3 = \begin{bmatrix} \frac{7}{8} \\ \frac{1}{8} \\ 0 \end{bmatrix}, x_4 = \begin{bmatrix} \frac{15}{16} \\ \frac{1}{16} \\ 0 \end{bmatrix}, \ \ldots, x_{n\to\infty} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$