PageRank

invented by Google co-founder Larry Page

Introduction

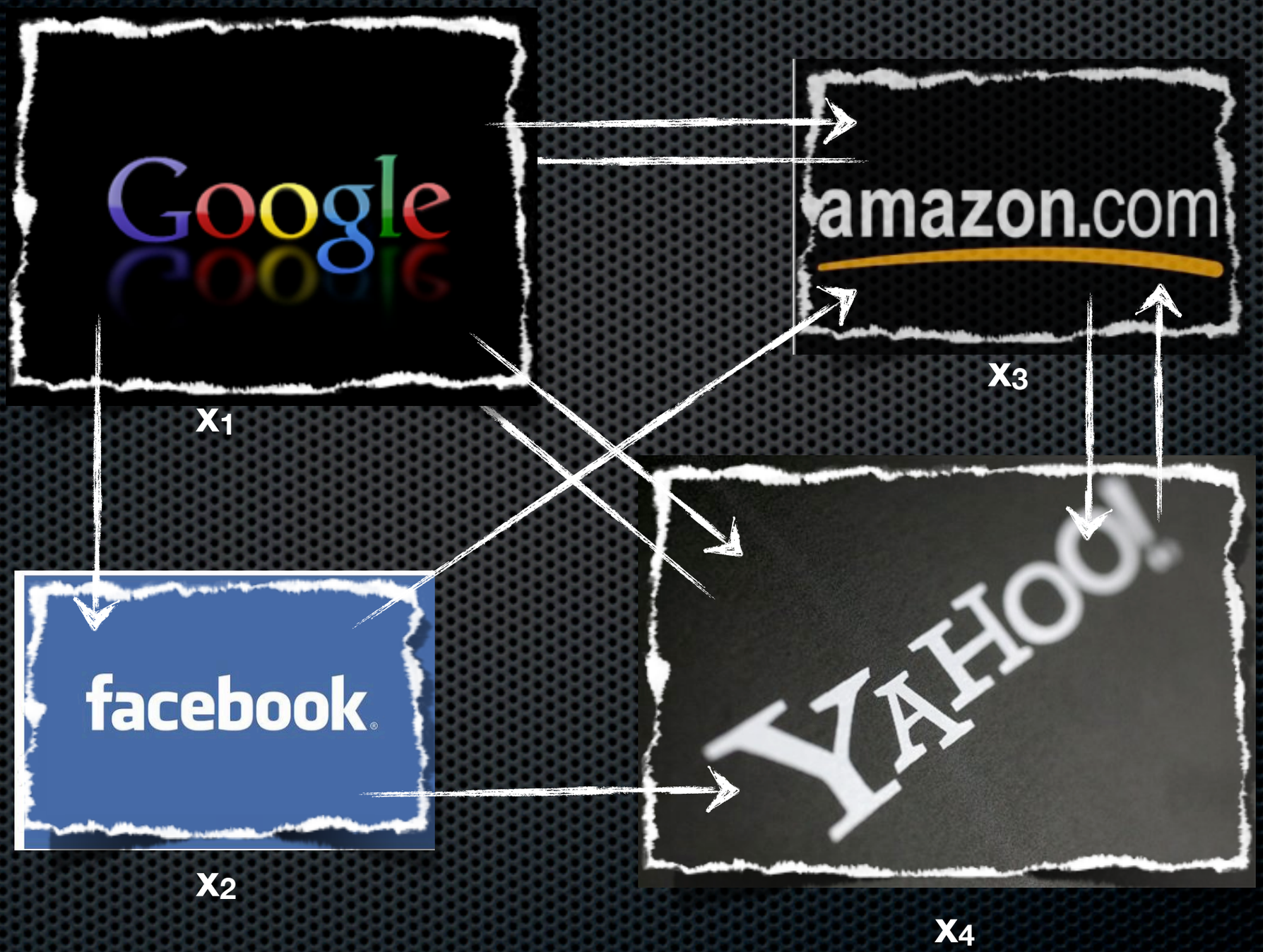
Page rank is an algorithm used by Google's search engine. It is named after the creator, Larry Page from Stanford University. This was published and implemented into Google's search engine in 1998 where shortly after, Page and Sergey Brin (a research partner) founded Google Inc. While this is just one of the many tools used in Google's search results, it provides a basis for all of the search tools.

Search Engine Basics

PageRank Explained

“PageRank relies on the uniquely democratic nature of the web by using its vast link structure as an indicator of an individual page's value. In essence, Google interprets a link from page A to page B as a vote, by page A, for page B. But, Google looks at more than the sheer volume of votes, or links a page receives; it also analyzes the page that casts the vote. Votes cast by pages that are themselves "important" weigh more heavily and help to make other pages "important".”

How To Rate Websites



Solving the Matrix Equation

$x_1 = x_3/1 + x_4/2$
$x_2 = x_1/3,$
$x_3 = x_1/3 + x_2/2 + x_4/2$
$x_4 = x_1/3 + x_2/2$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

Eigenvectors with eigenvalue 1 are all multiples of $[12\ \ 4\ \ 9\ \ 6]^T$

$x_1 = 12/31 \approx 0.387$
$x_2 = 4/31 \approx 0.129$
$x_3 = 9/31 \approx 0.290$
$x_4 = 6/31 \approx 0.194$

Google ≈ 0.387
Facebook ≈ 0.129
Amazon ≈ 0.290
Yahoo ≈ 0.194

Proposed Method

- Google's PageRank algorithm uses the basic Linear Algebra concept of eigenvectors to "rate" pages based on the importance of their incoming links.
- Eigenvectors are calculated using a standard procedure involving a vector's eigenvalues.
- By producing a normalized eigenspace basis, Google is able to determine how pages should be ordered in a search result.

Finding Eigenvalues

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$$

$$\begin{aligned} \det(A-\lambda I) &= \det\left(\begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix}\right) \\ &= (5-\lambda)^2 \\ &= (5-\lambda+3)\,(5-\lambda+3) \\ &= (8-\lambda)\,(2-\lambda) \end{aligned}$$

A's eigenvalues are 8 and 2

Finding Eigenvectors

For $\lambda 1 = 2$:

$$(A-2I)x = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow x1 = -x2$$

$$\Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix}, \quad B_{\lambda_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

For $\lambda 2 = 8$:

$$(A-8I)x = \begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow x1 = x2$$

$$\Rightarrow x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix}, \quad B_{\lambda_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$B_{\lambda i}$, A's eigenvectors, form a basis for the eigenspace of A

Conclusion

In conclusion, Google utilizes this Page Rank system in order to determine the importance of a web page. The weight of each page is determined by the number of “votes” or links to a certain page and then is recursively calculated utilizing the eigenvectors. This has become a powerful tool, useful in their search engine. It enhances the way search results are returned. As we can see by Google’s success, it is used quite effectively.

Acknowledgments

- http://en.wikipedia.org/wiki/Page_rank
- http://www.rose-hulman.edu/~bryan/googleFinalVersionFixed.pdf
- Quiz 9 on Beachboard (Professor Jen-Mei Chang of CSULB)
- Pictures Used in the Presentation:
  - Google Logo: http://img.thesun.co.uk/multimedia/archive/00571/google_logo_682_571408a.jpg
  - Google Dark Logo: http://userlogos.org/files/logos/mafi0z/Google.jpg
  - Yahoo! Logo: http://ceoworld.biz/ceo/wp-content/uploads/2009/08/yahoo-logo.jpg
  - Facebook Logo: http://www.enjoyfrance.com/images/stories/world/tech/Facebook-logo.jpg
  - Amazon Logo: http://mhpbooks.com/mobylives/wp-content/uploads/2008/12/amazon_logo.jpg
  - Bug: http://store.tidbitstrinkets.com/blog/wp-content/uploads/2009/10/Spider1.png
  - Cloud: http://www.tsl.state.tx.us/ld/projects/trc/2007/manual/craftillos/cloud.jpg