Linear Algebra Applied to Graph Theory

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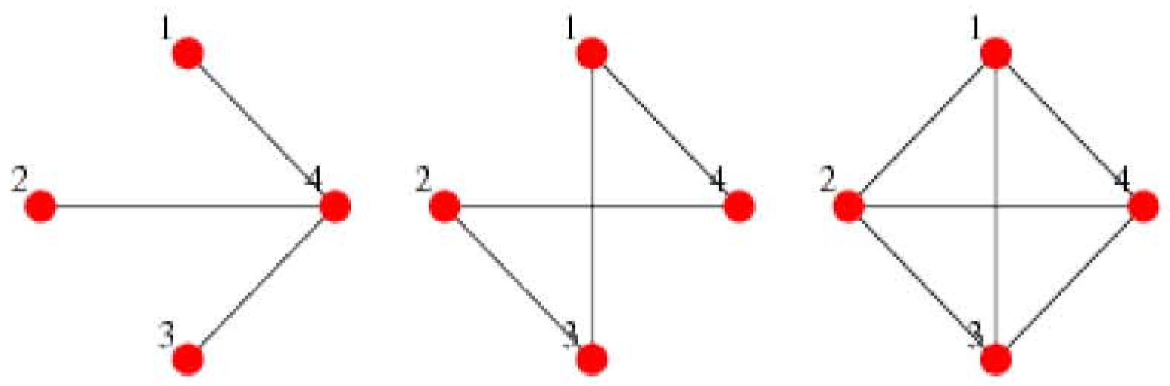
Introduction

Graph theory has been used for centuries to understand and solve numerous real-world problems, including everything from traffic flow to predictions of sports tournaments. Our focus is using graph theory to determine the best methods of traffic flow for any scenario, whether it is networking, electronic circuits, or delivery routes. To approach this problem, we consider the various matrix operations that are well-defined on the adjacency matrix, an established method of representing a graph mathematically.

Graph theory can be defined as the mathematical study of the properties of the formal mathematical structures called graphs. A graph is a collection of *nodes* and *edges*; a node represents a single entity, and an edge provides a connection between exactly two nodes. Edges can be directed, specifying a possibly non-symmetric relationship between two nodes. Nodes in a graph may have no edges (disconnected) or multiple edges. [1, 2]

The **adjacency matrix** of a simple graph is formed by first numbering the vertices in thee graph and then filling the matrix with the element (vi, vj) set to 1 if there is an edge between them (vi and vj are adjacent) and 0 otherwise. This convention requires that a simple graph with no loops yields an adjacency matrix with a 0-diagonal. [3]



$$\begin{pmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\1&1&1&0\end{pmatrix}\quad\begin{pmatrix}0&0&1&1\\0&0&1&1\\1&1&0&0\\1&1&0&0\end{pmatrix}\quad\begin{pmatrix}0&1&1&1\\1&0&1&1\\1&1&0&1\\1&1&1&0\end{pmatrix}$$

An adjacency matrix shows the layout of nodes in a graph.

Methods

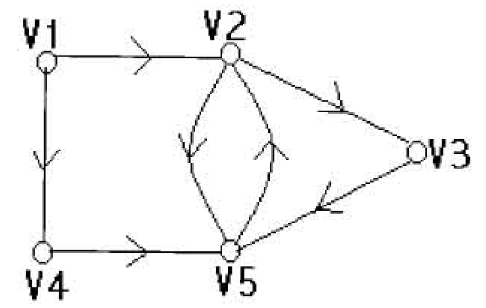
Our primary method of investigation was to apply various matrix operations to the adjacency matrices of sample graphs in order to determine whether any significant graph-wise transformation resulted.

In order to properly consider the various cases, cliques, connected graphs, disconnected graphs, and special cases in directed graphs were considered.

Results

Directed Graphs

Example: Given a directed graph G as follows

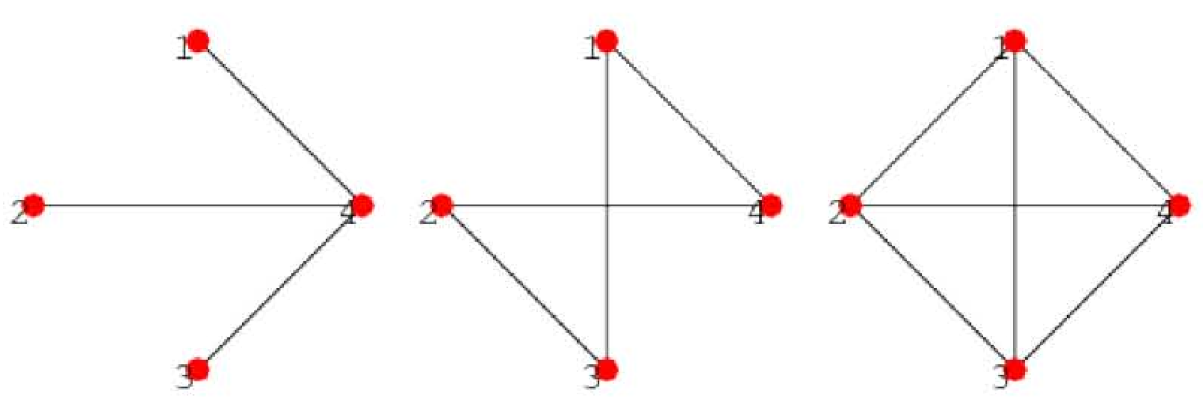


	V1	V2	V3	V4	V5
V1	0	1	0	1	0
V2	0	0	1	0	1
V3	0	0	0	0	1
V4	0	0	0	0	1
V5	0	1	0	0	0

From the chart above, the adjacency matrix for the directed graph is:

$$\begin{matrix}0&1&0&1&0\\0&0&1&0&1\\0&0&0&0&1\\0&0&0&0&1\\0&1&0&0&0\end{matrix}$$

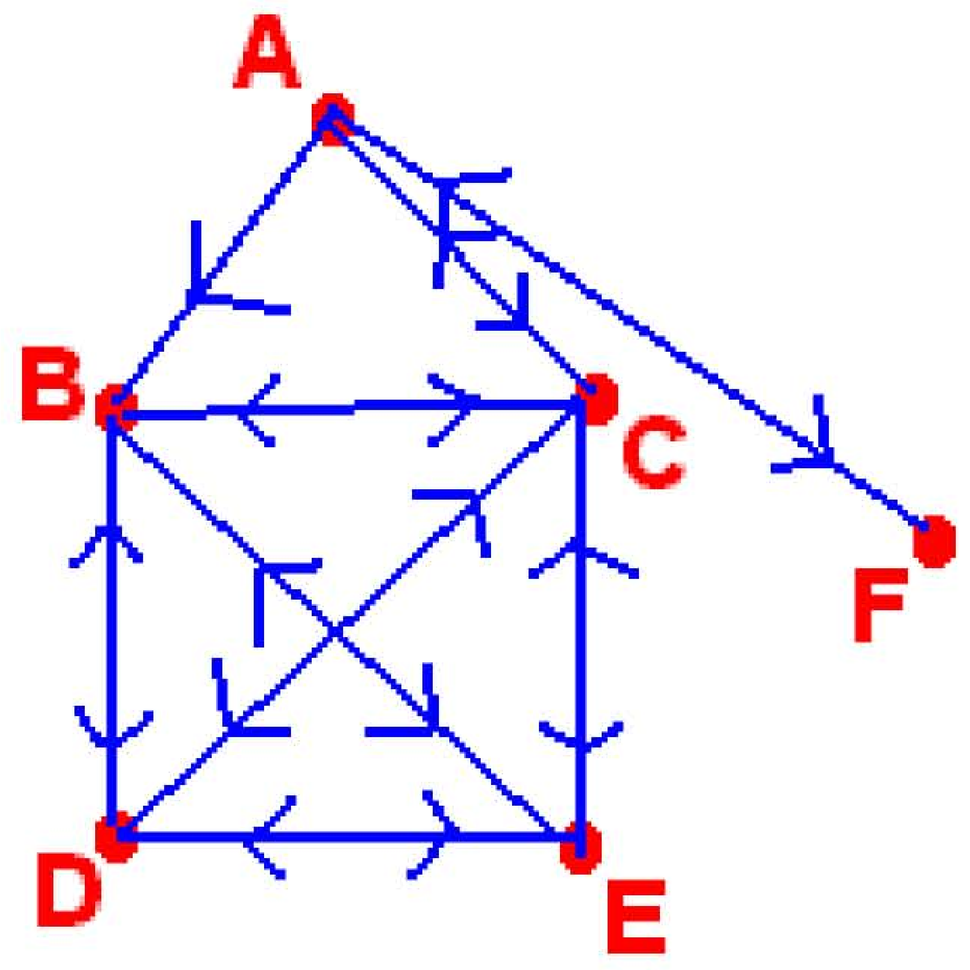
Incidence



$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\1&1&1\end{pmatrix}\quad\begin{pmatrix}1&1&0&0\\0&0&1&1\\1&0&1&0\\0&1&0&1\end{pmatrix}\quad\begin{pmatrix}1&1&1&0&0&0\\1&0&0&1&1&0\\0&1&0&1&0&1\\0&0&1&0&1&1\end{pmatrix}$$

Incidence matrices show the relationships between nodes and edges in a graph.

Clique



$$A(G)=\begin{pmatrix}0&1&0&1&1&1\\1&0&1&1&0&1\\1&1&0&0&1&1\\0&1&0&0&0&1\\1&0&1&1&0&0\\1&1&1&0&0&0\end{pmatrix}$$

$$S=\begin{pmatrix}0&1&0&0&1&1\\1&0&1&1&0&1\\0&1&0&0&1&1\\0&1&0&0&0&0\\1&0&1&0&0&0\\1&1&1&0&0&0\end{pmatrix}\qquad S^3=\begin{pmatrix}2&8&2&1&6&7\\8&4&8&4&2&6\\2&8&2&1&6&7\\1&4&1&0&2&2\\6&2&6&2&0&2\\7&6&7&2&2&4\end{pmatrix}$$

Determinant

The determinant of the adjacency matrix of a clique is the degree of every node.

Summary

In studying graph theory we find that we can use adjacency matrices and their components to solve real world applications involving flow rates. These applications range from path efficiency in delivery routes, network redundancy, and ranking predictions.

Using the power properties of an adjacency matrix we are able to see how many different paths there are from one point to another. As shown in our results the powers property of an adjacency matrix we see that the resultant matrix shows the number of paths between any two points.This property of an adjacency matrix is important in finding out the best delivery path for a delivery company. For instance, a company wants to know the number of paths from point A to point B. In using the adjacency matrix power property they determine that there is one 1-step path, and three 2-step paths and from that information the company can choose the most efficient paths.

In addition studying the adjacency matrix of a dominance directed graph we can predict the outcomes by examining the powers of the vertex in the graph. By adding the powers of the adjacency matrix we can identify the ranking of the outcomes of which is to happen first to last. As shown by our research and studies graph theory is very useful in real world applications. So if some ask when will I ever use that in the real world, now you have the answer.

Conclusions

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Acknowledgements

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