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EQUATIONS OF NATURE: A BRIEF LOOK AT FRACTAL GEOMETRY

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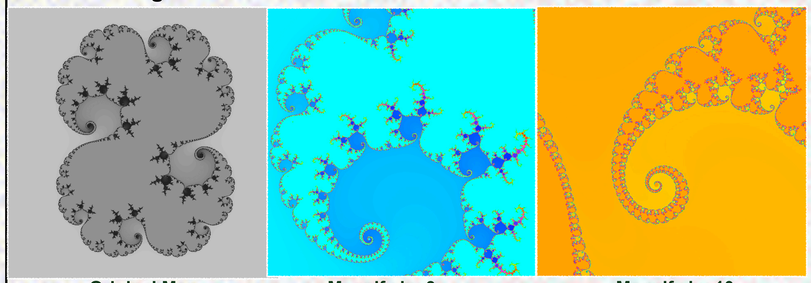
Introduction

A fractal is an image that is self-similar under ever increasing magnification. This means that as you zoom in on a fractal image, details emerge that mimic the original, or largest, framework of that image. Mother Nature provides us with beautiful examples of fractal patterns and structures, but it is what fractals can do for humans in science and research that is so fascinating.

Marek Dragon[1]



Original Map | Magnify by 3 | Magnify by 10

$z_{n+1} = z_c z_n + z_n^2$

Where z_c is $e^{2pi\ ri}$ and z_0 are the points in the complex image plane. "r" is an irrational, for the examples here it is 1 - (sqrt(2) - sqrt(3) + sqrt (5))/2.0

What makes Fractals useful? Matrices of course!

Fractal images are created by Affine Transformations. These are matrix based formulas that provide shifts and rotations to an image or curve. It is the existence of these algorithms that provide scientists, engineers, and artists with the tools to complete their work.

The very basic set up is as follows[2]:

$$y = A * x + T$$

where A is an n x n matrix, x is an n x 1 vector and T is an n x 1 vector.

The following equation serves to shift and transform the image:

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

The following is a matrix of rotation: where θ is the angle of rotation.

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Acknowledgments:

1)http://local.wasp.uwa.edu.au/~pbourke/fractals/marek/ , Submitted by Roger Bagula Graphics by Paul Bourke, August 2003
2)Ivo Marak - marak@sgi.felk.cvut.cz
3)http://mathforum.org/alejandre/workshops/fractal/fractal3.html
4)Lesmoir_Gordon, Nigel. Introducing Fractal Geometry. Lanham, Maryland: Totem Books, 2001. 3-171. Print.

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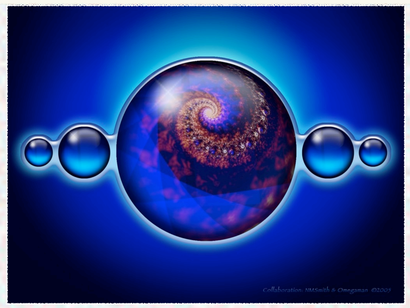
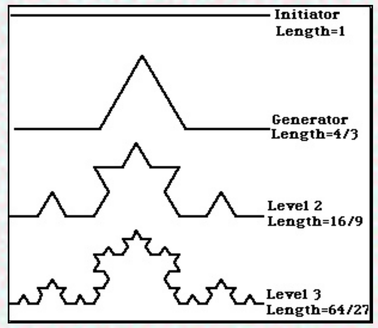


The popularity and shear beauty of fractal art has drawn many to find out more about what fractals are and how they relate to our everyday lives.

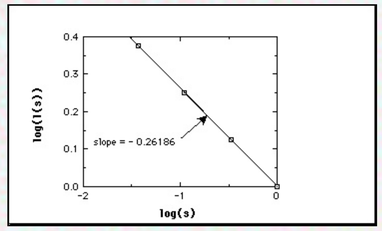
Analysis of the Koch curve allows us to see how fractals help us with coastal measurement.



What is the length of the Koch curve?

The length of the curve increases with each iteration. It has infinite length.

Coast lines are measured much in the same way. The smaller measuring device, the longer the coastline.



The fractal dimension is obtained from D = log(N)/log(r) D = log(4)/log (3) = 1.26.

Fun fact: Using fractals, we can see that Britain's coastline is not a 3 D coast, but more like 1.25. Almost the same dimension of the Koch Curve![4]

The Mathematics behind the Magic[3]

$$\begin{bmatrix} 1/3\cos 0 & -1/3\sin 0 \\ 1/3\sin 0 & 1/3\cos 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Scales figure by 1/3, rotates it by zero and shifts it by zero.

$$\begin{bmatrix} 1/3\cos 60 & -1/3\sin 60 \\ 1/3\sin 60 & 1/3\cos 60 \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix}$$

Scales figure by 1/3, rotates it 60° and shifts it by 1/3 in the x direction and zero in the y direction.

$$\begin{bmatrix} 1/3\cos(-60) & -1/3\sin(-60) \\ 1/3\sin(-60) & 1/3\cos(-60) \end{bmatrix} + \begin{bmatrix} 1/2 \\ \frac{\sqrt{3}}{6} \end{bmatrix}$$

Again, scales figure by 1/3, rotates it by 60° (in a new direction)! Shifts it by 1/2 in the x and √3/6 in the y direction.

$$\begin{bmatrix} 1/3\cos 0 & -1/3\sin 0 \\ 1/3\sin 0 & 1/3\cos 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By repeating Step one, we begin the series of scaling and rotation all again and our Koch Curve is developed.

Computer Applications

Fractal applications are most often associated with measurement of coastlines, growth patterns in nature, and modern works of art

Figure (a)

(See Merak Dragon to left). Fractals have also been used in computer graphics to simulate water, skin, and other textures. Computer based design software and hardware has typically been tailored towards producing precise drawings. See Figure (a). Two characteristics of a human sketch which do not occur in computer printed output from CAD packages are:

- The endpoints of lines tend to overshoot at an intersection
- Lines are not exactly straight but "wiggle"

One technique that modifies the drawing instructions on their way from the CAD software to the output device intercepts and changes the vectors to give the appearance of a human sketch before being forwarded to the plotter.

The approach used here is to modify the geometry of a 3D model itself. Nonintersecting lines are achieved by extending both ends of line segments by a random distance. The distance should be related to the line length. The wiggly line effect was generated by a well known technique from fractal geometry called "random midpoint displacement." It takes a straight line segment and generates a fractal curve by repeatedly splitting lines and perturbing their midpoint, the degree of wiggle is easily controlled by the user.

Convincing sketches can be created with these techniques. See Figure (b).

Figure (b)

Conclusion

The discovery of Affine Transformations and the iterated nature of the equations behind fractals has given us the power to further explore everything from chaos theory to graphic design. The ability to "marry" the finite qualities of math with the seemingly randomness of nature has brought science and art to an entirely new level of understanding and given us many more questions to explore.