Acknowledgments:
5/Marek Dragon

Introduction

A fractal is an image that is self-similar under ever increasing magnification. This means that as you zoom in on a fractal image, details emerge that mimic the original, or largest, framework of that image. Mother Nature provides us with beautiful examples of fractal patterns and structures, but it is what fractals can do for humans in science and research that is so fascinating.

The very basic set up is as follows:
\[ y = Ax + T \]
where \( A \) is an \( n \times n \) matrix, \( x \) is an \( n \times 1 \) vector and \( T \) is an \( n \times 1 \) vector.

The following equation serves to shift and transform the image:
\[
\begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
y_0 \\
x_0
\end{bmatrix}
\]

What makes Fractals useful? Matrices of course!

Fractal images are created by Affine Transformations. These are matrix based formulas that provide shifts and rotations to an image or curve. It is the existence of these algorithms that provide scientists, engineers, and artists with the tools to complete their work.

Analysis of the Koch curve allows us to see how fractals help us with coastal measurement.

The Mathematics behind the Magic

\[
\begin{bmatrix}
1/3 & 0 & -1/3 & 0 \\
1/3 & 0 & 1/3 & 0
\end{bmatrix}
\begin{bmatrix}
1/3 \cos 0 & -1/3 \sin 0 \\
1/3 \sin 0 & 1/3 \cos 0
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

Scales figure by 1/3, rotates it by zero and shifts it by zero.

\[
\begin{bmatrix}
1/3 \cos 60 & -1/3 \sin 60 \\
1/3 \sin 60 & 1/3 \cos 60
\end{bmatrix} + \begin{bmatrix}
1/2 \\
0
\end{bmatrix}
\]

Scales figure by 1/3, rotates it 60° and shifts it by 1/3 in the x direction and zero in the y direction.

Again, scales figure by 1/3, rotates it by 60° (in a new direction) Shifts it by 1/2 in the x and 3/8 in the y direction.

By repeating Step one, we begin the series of scaling and rotation all again and our Koch Curve is developed.

Conclusion

The discovery of Affine Transformations and the iterated nature of the equations behind fractals has given us the power to further explore everything from chaos theory to graphic design. The ability to “marry” the finite qualities of math with the seemingly randomness of nature has brought science and art to an entirely new level of understanding and given us many more questions to explore.