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M247
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Fractals

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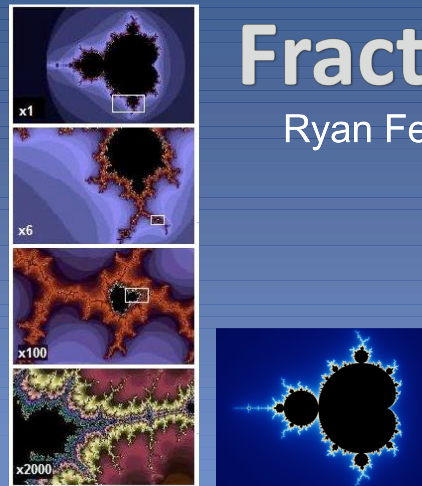
What are Fractals?

A fractal is a complex, geometric shape which has visible details under a variety of scales of magnification. Theoretically, a fractal has patterns under all scales of magnification. The diminishing pattern of fractals are approximately, or directly, self-similar.

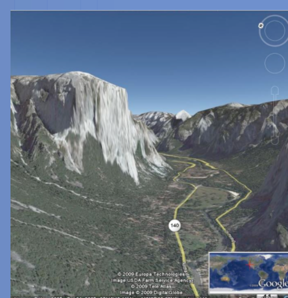


Mandelbrot Set

Use of Fractals

Most applications are used by scientists and engineers to describe complex patterns of the world:

Fractal Landscaping- Fractals can be used to form an artificial terrain. Virtual globe programs, like Google Earth, now have three dimensional mapping in order to better visualize landscape and elevation.



Google Earth 5.0

Sharper Measurements- More precise measurements can be used to measure coastlines. From a satellite view, if one were to zoom into the coastline of Britain, there would be more details of the coastline visible then the initial viewpoint. Therefore, bays or harbors that are smaller than the measurement scale might not be taken into account. Benoît Mandelbrot, a mathematics professor at Yale and the creator of the Mandelbrot Set, stated that the coastline of Britain would measure infinity as the measurement of scale approaches zero.

How long is the coast of Britain?

Image Compression- Using fractal image compressing programs, many mathematical descriptions called “fractal codes” can be taken from a bitmap image. These “fractal codes” are then used to recreate the original bitmap image, but with independent resolution. The fractal coded image can fit any screen size without becoming pixilated.

Brain Patterns- The human brain has complex wiring circuits which have a fractal shape. Perhaps fractals might help scientists discover how the human brain works.

Iterated Function Systems

Iterated function systems are a set of affine transformations used to generate fractal patterns. The set of transformations are looped many times in order to produce fractals with self-similarity.

$$\begin{bmatrix} xf \\ yf \end{bmatrix} = c * \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

Shape Transformed | Scalar | Rotational Matrix | Initial Shape | Shifter

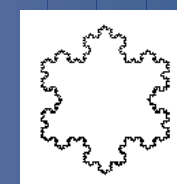
Koch Curve

$$\begin{bmatrix} x1 \\ y1 \end{bmatrix} = \frac{1}{3} * \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

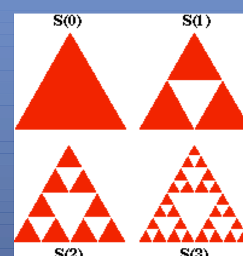
$$\begin{bmatrix} x2 \\ y2 \end{bmatrix} = \frac{1}{3} * \begin{bmatrix} \cos(\pi/3) & -\sin(\pi/3) \\ \sin(\pi/3) & \cos(\pi/3) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x3 \\ y3 \end{bmatrix} = \frac{1}{3} * \begin{bmatrix} \cos(-\pi/3) & -\sin(-\pi/3) \\ \sin(-\pi/3) & \cos(-\pi/3) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ \sqrt{3}/6 \end{bmatrix}$$

$$\begin{bmatrix} x4 \\ y4 \end{bmatrix} = \frac{1}{3} * \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \end{bmatrix}$$



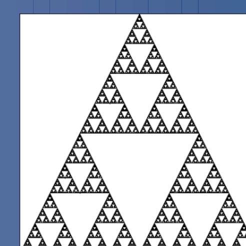
Sierpinski Triangle



$$\begin{bmatrix} x1 \\ y1 \end{bmatrix} = \frac{1}{2} * \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x2 \\ y2 \end{bmatrix} = \frac{1}{2} * \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x3 \\ y3 \end{bmatrix} = \frac{1}{2} * \begin{bmatrix} \cos(0) & -\sin(0) \\ \sin(0) & \cos(0) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ \sqrt{3}/4 \end{bmatrix}$$



Conclusion

Fractals are a relatively new concept. Applications are sill being created as we speak. Perhaps, in the next 50 years, the use of fractals will create remarkable applications.

References

Riddle, Larry. "Classic Iterated Function Systems." *Anges Scott College* Web.10 May 2009. <http://ecademy.agnesscott.edu/~lriddle/ifs/ifs.htm>.
Clarke, Arthur C.. *The Colours of Infinity*. Singapore: Clear Press, 2006. Print.