The Leontief Input- Output Model

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Introduction

Wassily Leontief created an input-output model for economics. His model is a basis for more models currently being used in many parts of the world. This model can be applied to any size economy from a small business to the whole world. The main goal of the Leontief Input-Output model is to balance the total amount of goods produced to the total demand for that production.

Inputs Consumed per Unit of Output

Purchased from:	Manufacturing	Agriculture	Services
Manufacturing	.50	.40	.20
Agriculture	.20	.30	.10
Services	.10	.10	.30

$$C = \begin{bmatrix} .50 & .40 & .20 \\ .20 & .30 & .10 \\ .10 & .10 & .30 \end{bmatrix}$$

$$\mathbf{x} = C\mathbf{x} + \mathbf{d}$$
$$I\mathbf{x} - C\mathbf{x} = \mathbf{d}$$
$$(I-C)\mathbf{x} = \mathbf{d}$$

$$\underset{\mathbf{x}}{\text{amount produced}} = \text{intermediate demand} + \underset{\mathbf{d}}{\text{final demand}}$$

$$I-C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .5 & .4 & .2 \\ .2 & .3 & .1 \\ .1 & .1 & .3 \end{bmatrix} = \begin{bmatrix} .50 & -.40 & -.20 \\ -.20 & .70 & -.10 \\ -.10 & -.10 & .70 \end{bmatrix}$$

$$\begin{bmatrix} .50 & -.40 & -.20 & 50 \\ -.20 & .70 & -.10 & 30 \\ -.10 & -.10 & .70 & 20 \end{bmatrix} \sim \ldots \sim \begin{bmatrix} 1 & 0 & 0 & 226 \\ 0 & 1 & 0 & 119 \\ 0 & 0 & 1 & 78 \end{bmatrix}$$

Method

The method that the Leontief Input- Output Model uses is the matrix. The nation's economy is divided into **n** sectors that produce goods or services. There exists a production vector **x** which is the output. There is also the final demand vector **d** which is the value of the goods and services demanded. For each sector there is a unit consumption vector which lists the inputs needed per unit of output of the sector. The consumption vectors together form a consumption matrix C.

Results

From the example using the Leontief Input-Output model one can get results when finding out exactly how much production should be made. In this example the demand was manufacturing- 50, agriculture- 30, and services- 20.

1. Augmented matrix C.
2. Subtract C from identity matrix I.
3. Row reduce (I-C) to find **x** , the production that will satisfy the demand.

In this example, the production level would be manufacturing- 226, agriculture- 119, and services- 78.

- Other approaches
  production vector: $\mathbf{x} = (I-C)\mathbf{d}$
  Unique solution $\mathbf{x} = C\mathbf{x} + \mathbf{d}$

Summary

The Leontief Input-Output model is a valuable tool for economics. It is able to make predictions into the future. As one can tell it uses linear algebra concepts. Such concepts are matrices, the inverse concept, and the row reduced echelon form. Overall, it can be applied to different economic settings and will always give valuable results.

Acknowledgements

1. Lay, David. *Linear Algebra and Its Applications*. Third. MA: Pearson Education, Inc., 2006. Print.

2. "Module for the Leontief Economic Model." Leontief Economic Model Web. 26 Apr 2009. http://math.fullerton.edu/mathews/n2003/LeontiefModelMod.html.