Image Compression

Michael Combs

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Asst. Prof. Jen-Mai Chang

California State University
Long Beach

Introduction

The widespread use of digital cameras with ever higher resolution has place increasing demands on storage technology. A key method to reduce this demand is known as image compression. This project will describe a technique known as the "Harr Wavelet Transform".

What is an Image?

An image is made up of discrete points called pixels. For a greyscale image, each pixel will contain a value for its intensity. For a 8 bit image, the values of intensity will vary from 0 (black) to 255 (white) as shown in Figure 1.

Figure 1. Range of tones in grayscale image (0 to 248 in increments of 8)

Image as a Matrix

A grayscale image can be described as a two dimension matrix where the values in the matrix reflect the intensity of each pixel.

How it Works

The compression technique can be more easily shown by applying it to the first row of an 8X8 matrix

- Replace the first 4 elements in the row with the averages of the 4 pairs. The last four elements are created by subtracting the first element of each pair from its corresponding average.

- Repeat the process again, only this time using only the first two pairs of elements.

- Repeat the process one more time using only the first pair of elements

The results of each step being applied to the first row of an 8X8 matrix is shown in Table 1. Once the process has been completed on the rows, it is then applied to the columns.

Original:	64	48	16	32	56	56	48	24
Step 1:	56	24	56	36	8	-8	0	12
Step 2:	40	46	16	10	8	-8	0	12
Step 3:	43	-3	16	10	8	-8	0	12

Table 1. Harr wavelet transform being applied to row 1 of 8X8 matrix one step at a time. Pair averages in blue, first value of each pair minus average in red.

Transformation Using Linear Algebra

The set of row operations can be simplified by combining the steps into a single right matrix multiplication using matrix W (Table 2).

1/8	1/8	1/4	0	1/2	0	0	0
1/8	1/8	1/4	0	-1/2	0	0	0
1/8	1/8	-1/4	0	0	1/2	0	0
1/8	1/8	-1/4	0	0	-1/2	0	0
1/8	-1/8	0	1/4	0	0	1/2	0
1/8	-1/8	0	1/4	0	0	-1/2	0
1/8	-1/8	0	-1/4	0	0	0	1/2
1/8	-1/8	0	-1/4	0	0	0	-1/2

Table 2. Harr wavelet row transformation as a matrix

The column transform is accomplished by left multiplication using W^T, so the complete transform is $T = W^T*A*W$, where A is the original matrix.

The result is an overall average in the upper left hand corner with differences (detail coefficients) in the remainder of the matrix.

To restore the image, reverse the process: $Acomp = (W^T)^{-1}*N*W^{-1}$

Note that the columns of the matrix W shown in Table 2 are orthogonal. If the columns are normalized as shown in Table 3, then $W^T=W^{-1}$, and the reversal process is simplified into $Acomp = W*N*W^T$.

$\frac{1}{\sqrt{8}}$	$\frac{1}{\sqrt{8}}$	$\frac{1}{2}$	0	$\frac{1}{\sqrt{2}}$	0	0	0
$\frac{1}{\sqrt{8}}$	$\frac{1}{\sqrt{8}}$	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{2}}$	0	0	0
$\frac{1}{\sqrt{8}}$	$\frac{1}{\sqrt{8}}$	$-\frac{1}{2}$	0	0	$\frac{1}{\sqrt{2}}$	0	0
$\frac{1}{\sqrt{8}}$	$\frac{1}{\sqrt{8}}$	$-\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{2}}$	0	0
$\frac{1}{\sqrt{8}}$	$-\frac{1}{\sqrt{8}}$	0	$\frac{1}{2}$	0	0	$\frac{1}{\sqrt{2}}$	0
$\frac{1}{\sqrt{8}}$	$-\frac{1}{\sqrt{8}}$	0	$\frac{1}{2}$	0	0	$-\frac{1}{\sqrt{2}}$	0
$\frac{1}{\sqrt{8}}$	$-\frac{1}{\sqrt{8}}$	0	$-\frac{1}{2}$	0	0	0	$\frac{1}{\sqrt{2}}$
$\frac{1}{\sqrt{8}}$	$-\frac{1}{\sqrt{8}}$	0	$-\frac{1}{2}$	0	0	0	$-\frac{1}{\sqrt{2}}$

Table 3. Harr wavelet matrix using orthonormal columns

How is this Compression?

The size of T can now be reduced by eliminating the smallest detail coefficients (replace with zero). The reduced version of T (call it N) is the compressed file. See Figures 2-4 for examples of different levels of compression applied to a 1024X1024 image.

Figure 2. Original Image (5.5% zeroes in N matrix)

Figure 3. Threshold = 10 (91.6% zeroes in N matrix)

Figure 4. Threshold = 30 (97.4% zeroes in N matrix)

References

"Image Compression", Mike Oremus and Ottopaskal Rice, 5 December 2000.
"Plotting and Scheming with Wavelets", Colm Mulcahy, Dept. of Mathematics, Spelman College.