Relationships between Individuals in a Group

Introduction to Graph Theory

- A graph is a collection of points called *vertices* joined by lines called *edges*. (See Figure 1)
- This graph can be turned into an adjacency matrix M by assigning an entry a_{ij} for the path from one point to another. If there is an edge connecting the two points, the entry is assigned a *1*. If not, a *0*. (See Figure 2)
- By squaring the adjacency matrix M, we can find two step paths between 2 points. (See Figure 3)
  - For example: Between points P_1 and P_9 there are 2, 2-step paths, P_1-P_6-P_9 and P_1-P_8-P_9. (See Figure 1A)
- By using the formula $M + M^2$, we can find the total number of 1-step and 2-step paths from one point to another. (See Figure 4)

Application to Sociology

- Sociologists study the relationships between individuals in a group. Somehow each of these individuals are connected to each other, either directly or indirectly.
- A group of individuals can be simplified into a graph of points and connecting lines to create a graph. The vertices represent the individual people and the edges represent their connection.
- For example, Figure 1 may represent the ties between family members.

P_1 – Paul	(Nancy's husband)
P_2 – Nancy	(Paul's wife)
P_3 – James	(Paul and Nancy's son)
P_4 – Molly	(James' wife)
P_5 – Ralph	(James and Molly's son / Annie's cousin)
P_6 – Frank	(Paul's co-worker)
P_7 – Lindsay	(Frank's Wife / Nancy's friend)
P_8 – Charlie	(Frank and Lindsay's son / James' brother)
P_9 – Danielle	(Charlie's wife)
P_{10} – Annie	(Charlie and Danielle's daughter)

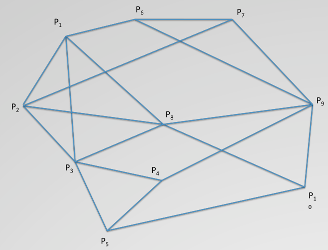

Figure 1

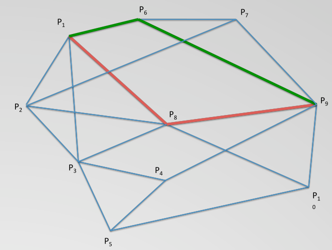

Figure 1A

P_1 – Paul (Nancy's husband)
P_2 – Nancy (Paul's wife)
P_3 – James (Paul and Nancy's son)
P_4 – Molly (James' wife)
P_5 – Ralph (James and Molly's son / Annie's cousin)
P_6 – Frank (Paul's co-worker)
P_7 – Lindsay (Frank's Wife / Nancy's friend)
P_8 – Charlie (Frank and Lindsay's son / James' brother)
P_9 – Danielle (Charlie's wife)
P_{10} – Annie (Charlie and Danielle's daughter)

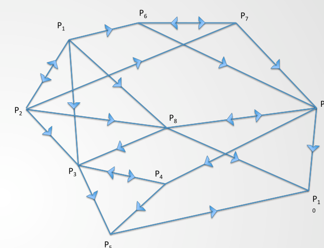

Figure 1B

Figure 2

M =

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
P_1	0	1	1	0	0	1	0	1	0	0
P_2	1	0	1	0	0	0	1	1	0	0
P_3	0	0	0	1	1	0	0	0	0	0
P_4	0	0	1	0	1	0	0	0	0	0
P_5	0	0	0	0	0	0	0	0	0	1
P_6	0	0	0	0	0	0	1	0	1	0
P_7	0	0	0	0	0	1	0	0	1	0
P_8	0	0	1	0	0	0	0	0	1	1
P_9	0	0	0	1	0	0	0	1	0	1
P_{10}	0	0	0	0	0	0	0	0	0	0

Figure 3

M^2 =

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
P_1	1	0	2	1	1	0	2	1	2	1
P_2	0	1	2	1	1	2	0	1	2	1
P_3	0	0	1	0	1	0	0	0	0	1
P_4	0	0	0	1	1	0	0	0	0	1
P_5	0	0	0	0	0	0	0	0	0	0
P_6	0	0	0	1	0	1	0	1	1	1
P_7	0	0	0	1	0	0	1	1	1	1
P_8	0	0	0	2	1	0	0	1	0	1
P_9	0	0	2	0	1	0	0	0	1	1
P_{10}	0	0	0	0	0	0	0	0	0	0

Figure 4

$M + M^2$ =

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}		
P_1	1	1	3	1	1	1	2	2	2	1	=	15
P_2	1	1	3	1	1	2	1	2	2	1	=	15
P_3	0	0	1	1	2	0	0	0	0	1	=	5
P_4	0	0	1	1	2	0	0	0	0	1	=	5
P_5	0	0	0	0	0	0	0	0	0	1	=	1
P_6	0	0	0	1	0	1	1	1	2	1	=	7
P_7	0	0	0	1	0	1	1	1	2	1	=	7
P_8	0	0	1	2	1	0	0	1	1	2	=	8
P_9	0	0	2	1	1	0	0	1	1	2	=	8
P_{10}	0	0	0	0	0	0	0	0	0	0	=	0

Ranking

Ranking
P_1 – Paul
P_2 – Nancy
P_8 – Charlie
P_9 – Danielle
P_6 – Frank
P_7 – Lindsay
P_3 – James
P_4 – Molly
P_5 – Ralph
P_{10} – Annie

Note that from the original adjacency matrix (M), Paul has influence over Frank, Nancy, James, and Charlie. However, by squaring the matrix (M), we find that Paul has 2-stage influence over everyone.

Figure 5

Application to Sociology

- By drawing directions to the connecting lines, we mark the dominance of one individual over another. (See Figure 1B)
- Just like with the graph, this graph can be represented by an adjacency matrix M.
  - An entry of 1 represents the dominance of the *i*-component over the *j*-component.
  - An entry of 0 shows that there is no direct dominance between those two individuals.
- Again, when this graph is squared, we can see the 2-step connections of dominance between 2 individuals. (See Figure 3)
  - For example, Paul is dominant over Danielle through both Frank and Charlie, so this is represented with a 2.
- By using the formula $M + M^2$, we find the total 1-step and 2-step paths between the individuals in the group. (See Figure 4)
- We can also rank the individuals by dominance by adding the sum of their rows in the matrix $M + M^2$. (See Figure 5)

From Linear Algebra to Sociology

- Using the fundamentals of Linear Algebra and a formula from Graph Theory, we are able to graph and determine connections between individuals in a group more clearly.
- This method can be used in many other situations:
  - Which sports team is most likely to do well next season?
    - Using information from previous games, we can determine the likelihood of one team to beat another.
  - Who is the most influential employee in an office?
    - Is it the CEO or the mail clerk? Depending on the company and the work environment, it could be either.

Acknowledgements

- University of Ottawa

http://aix1.uottawa.ca/~jkhoury/app.htm