Cramer's Rule, Volume, and Linear Transformations

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How is Cramer's Rule Used?

Cramer's rule as an application to see geometrically what happens when a change is made to one of the x's in the Ax=B formula. In engineering, Cramer's rule is used to analyze differential equations by creating parameters. The geometric interpretation of determinants is that they are used to find area and volume given two or three vectors. The underlying principle is to use determinants to figure out area, volume, or the parameters of x given vectors.

Cramer's Rule

Let A be an invertible n x n matrix. For any b in R^n, the unique solution x of Ax=b has entries given by

$$x_i = \frac{\det A_i(b)}{\det A}$$ i = 1, 2, …, n

Example

Use Cramer's rule to solve the system.

$$4x_1 + x_2 = 6$$
$$3x_1 + x_2 = 5$$

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

$$A_1(b) = \begin{bmatrix} 6 & 1 \\ 5 & 1 \end{bmatrix}$$

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{1}{1}$$

$$A_2(b) = \begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix}$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{2}{1}$$

Application to engineering

Using parameters is how in engineering determinants can come in handy.
For example:

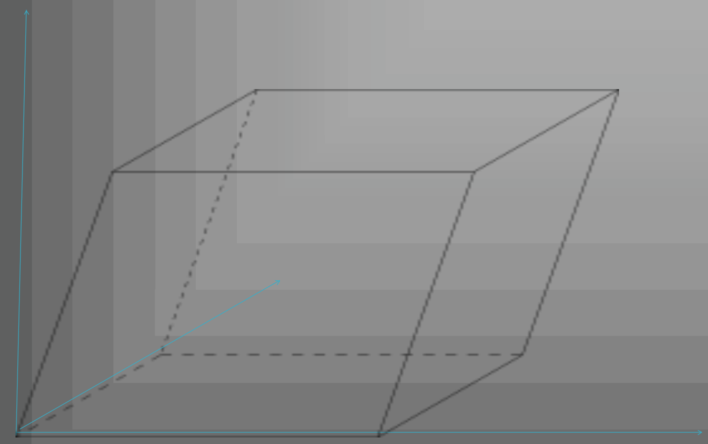
$$2sx_1 - 2x_2 = 2$$
$$x_1 + 2sx_2 = 5$$
$$A = \begin{bmatrix} 2s & -2 \\ 1 & 2s \end{bmatrix}$$
$$\det A = 4s^2 - 2$$
$$A_1(b) = \begin{bmatrix} 1 & -2 \\ 5 & 2s \end{bmatrix}$$
$$\det A_1(b) = 2s - 10$$
$$A_2(b) = \begin{bmatrix} 2s & 1 \\ 1 & 5 \end{bmatrix}$$
$$\det A_2(b) = 10s - 1$$
$$x_1 = \frac{2s - 10}{4s^2 - 2}$$
$$x_2 = \frac{10s - 1}{4s^2 - 2}$$

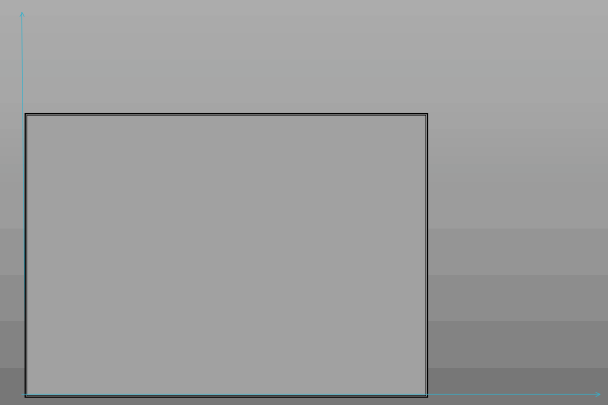
- How the system will behave when there is one parameter that can change?

At s =root 2 x has a unique solution for both x’s

What does a determinant mean geometrically?
Area and Volume

If A is an n x n matrix, the area of the parallelogram determined by the columns of is |det A|. If A is a 3 X 3 matrix, the volume of the parallelepiped determined by the columns of A is |det A|.





Using determinants is a way to solve for the volume of a 2x 2 matrix and a three by three matrix in space. determinants are a way to create parameters for a certain vector space to determine where the unique solutions of the values of x fall. These unique solutions can be found using the formula given by Cramer's rule, which turns out to be an easy way of finding a solution without solving for every single variable in the equation.