Linear Algebra Used in Computer Graphics

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INTRODUCTION

Computer graphics have quickly become a part of everyday life. Used in things such as video game, movies, aircraft design and educational presentations, computer graphics have become the building blocks of the modern world (Lay, p 158).

What are computer graphics?

Computer graphics, according to our text, are images displayed or animated on a computer screen (Lay, p 158).

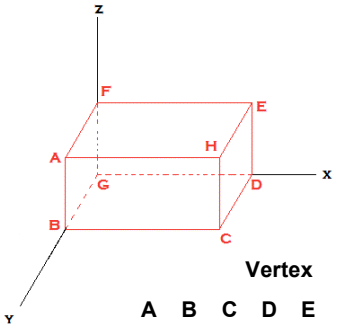
How do computer graphics relate to linear algebra?

When creating computer graphics, data is stored in matrices. The vertices of the image are stored by coordinates in the matrices and are manipulated by multiplying different matrices together and observing the effect of $x \rightarrow Ax$

How do I multiply matrices?

$$D=\begin{bmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{bmatrix} \qquad E=\begin{bmatrix} c_{11} & d_{12} \\ c_{21} & d_{22} \end{bmatrix}$$

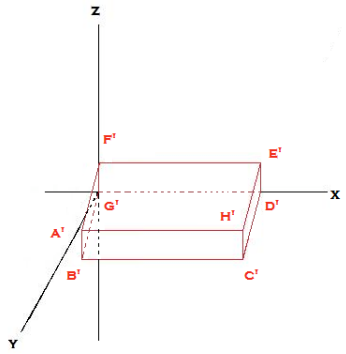
$$DE=\begin{bmatrix} a_{11}c_{11}+b_{12}c_{21} & a_{11}d_{12}+b_{12}d_{22} \\ a_{21}c_{11}+b_{22}c_{21} & a_{21}d_{12}+b_{22}d_{22} \end{bmatrix}$$



Vertex

	A	B	C	D	E	F	G	H
x-coordinate	0	0	2	2	2	0	0	2
y-coordinate	1	1	1	0	0	0	0	1
z-coordinate	1	0	0	0	1	1	0	1

Multiply A by B= $\begin{bmatrix} 1 & 1/8 & 0 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1/4 \end{bmatrix}$

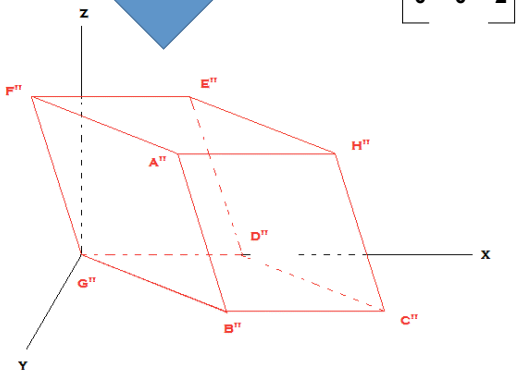


We see that AB=

Vertex

	A'	B'	C'	D'	E'	F'	G'	H'
x-coordinate	1/8	1/8	17/8	2	2	0	0	17/8
y-coordinate	3/2	1	1	0	1/2	1/2	0	3/2
z-coordinate	1/4	0	0	0	1/4	1/4	0	1/4

Multiply A by C= $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$



We see that AC=

Vertex

X-coordinate	2	2	4	2	2	0	0	4
Y-coordinate	2	1	1	0	1	1	0	2
Z-coordinate	2	0	0	0	2	2	0	2

SUMMARY

When multiplying A by B the graphic becomes much shorter on the z-axis and on the y-axis, changing the orientation on the x-axis as well. When multiplying A by C the graphic becomes much taller on the z-axis and longer on the y-axis. You can also see the change in orientation on the x-axis, making the graphic have a much more slanted appearance.
When designing video games or even computer animation, the designers will use this same basic principle to change parts of the graphic to fit the designers needs.

Sources

Lay, David C. Linear Algebra and its Applications.

http://www.math.harvard.edu/archive/21b_fall_04/exhibits/imageprocessing/index.html