



# Battle of Performance: Flipped vs. Conventional

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## Motivations

This is the first math class to use a flipped model at CSULB, so we wanted to compare and contrast student performance in the conventional classroom model versus the flipped classroom model.

## Introduction

We will be using Markov Chains to analyze grade trends between both flipped and conventional Linear Algebra classes.

## Markov Chains

A Markov Chain is a method used to predict future outcomes in a trending data set.

The Markov Chain is generalized by the equation  $Px_i = x_{i+1}$ .  $P$  is a stochastic matrix, which depicts data migration between domain values,  $x_i$  is the last known data set, and  $x_{i+1}$  is the data set to be predicted.

## Method

We used grade data from both class models to track the performance of individual students throughout the class.

| Cumulative Grade |        |        | Test Grades |        |        |
|------------------|--------|--------|-------------|--------|--------|
| Test 1           | Test 2 | Test 3 | Test 1      | Test 2 | Test 3 |
| A                | A      | A      | A           | B      | A      |
| B                | B      | B      | B           | C      | A      |
| B                | B      | B      | B           | C      | A      |
| B                | B      | B      | C           | A      | A      |
| A                | B      | B      | A           | B      | B      |
| B                | B      | B      | B           | B      | B      |
| B                | B      | B      | C           | B      | B      |

We used this information to create stochastic matrices for test grades and overall grades for both the flipped and conventional models. This will serve as the matrix  $P$  in the Markov Chain.

## Probability Vectors and the Stochastic Matrix

| Data   |        |
|--------|--------|
| Test 1 | Test 2 |
| A      | A      |
| A      | A      |
| B      | A      |
| B      | A      |
| A      | B      |
| B      | C      |
| B      | C      |
| C      | A      |
| A      | B      |
| B      | B      |
| C      | B      |
| C      | B      |
| A      | C      |
| C      | C      |
| C      | C      |

Note in this example that 5 students scored an A on test 1. Of those 5, 2 scored A's on test 2, 2 scored B's, and 1 scored a C.

This data is represented with the vector  $\langle 2/5, 2/5, 1/5 \rangle$ .

This type of vector, where all of the entries are nonzero, and add up to 1, is called a probability vector.

Similar vectors are made for the B & C grades, and these vectors are placed in a matrix

A matrix whose columns are made up of probability vectors is known as a Stochastic Matrix.

This example is based on the above data

| Stochastic Matrix |   |
|-------------------|---|
|                   | A B C   |
| A                 | $\begin{bmatrix} 2/5 & 2/5 & 1/5 \end{bmatrix}$ |
| B                 | $\begin{bmatrix} 2/5 & 1/5 & 2/5 \end{bmatrix}$ |
| C                 | $\begin{bmatrix} 1/5 & 2/5 & 2/5 \end{bmatrix}$ |

The experimental Stochastic Matrices were derived from three data points representing test scores and cumulative grades. The average of these matrices were used in the final calculations.

| Stochastic Matrices Based on Flipped Classroom Data |       |      |       |      |         |      |      |      |      |      |      |      |
|---|-------|------|-------|------|---------|------|------|------|------|------|------|------|
| Cumulative Grades                                   | 1 → 2 |      | 2 → 3 |      | Average |      |      |      |      |      |      |      |
|   | A     | B    | C     | F    | A       | B    | C    | F    | A    | B    | C    | F    |
| A   | 0.67  | 0.09 | 0.00  | 0.00 | 0.80    | 0.00 | 0.00 | 0.00 | 0.73 | 0.05 | 0.00 | 0.00 |
| B   | 0.33  | 0.82 | 0.25  | 0.00 | 0.20    | 0.85 | 0.20 | 0.00 | 0.27 | 0.83 | 0.23 | 0.00 |
| C   | 0.00  | 0.09 | 0.63  | 0.50 | 0.00    | 0.15 | 0.60 | 0.00 | 0.00 | 0.12 | 0.61 | 0.25 |
| D   | 0.00  | 0.00 | 0.13  | 0.50 | 0.00    | 0.00 | 0.20 | 0.60 | 0.00 | 0.00 | 0.16 | 0.55 |
| F   | 0.00  | 0.00 | 0.00  | 0.00 | 0.00    | 0.00 | 0.00 | 0.40 | 0.00 | 0.00 | 0.00 | 0.80 |

| Test Grades | 1 → 2 |      | 2 → 3 |      | Average |      |      |      |      |      |      |      |
|-------------|-------|------|-------|------|---------|------|------|------|------|------|------|------|
|             | A     | B    | C     | F    | A       | B    | C    | F    | A    | B    | C    | F    |
| A           | 0.40  | 0.25 | 0.10  | 0.00 | 0.67    | 0.09 | 0.20 | 0.00 | 0.53 | 0.17 | 0.15 | 0.00 |
| B           | 0.40  | 0.38 | 0.40  | 0.00 | 0.17    | 0.36 | 0.30 | 0.00 | 0.28 | 0.37 | 0.35 | 0.00 |
| C           | 0.20  | 0.38 | 0.30  | 0.50 | 0.00    | 0.27 | 0.20 | 0.00 | 0.10 | 0.32 | 0.25 | 0.09 |
| D           | 0.00  | 0.00 | 0.00  | 0.50 | 0.00    | 0.18 | 0.00 | 0.29 | 0.00 | 0.09 | 0.00 | 0.25 |
| F           | 0.00  | 0.00 | 0.20  | 0.00 | 0.17    | 0.09 | 0.30 | 1.00 | 0.08 | 0.05 | 0.25 | 0.50 |

## Calculations

$$x_i = \begin{bmatrix} \# \text{ of A's} \\ \# \text{ of B's} \\ \# \text{ of C's} \\ \# \text{ of D's} \\ \# \text{ of F's} \end{bmatrix}$$

For  $x_i$ , we used the last known grade distribution for the flipped class and placed them into a vector with each letter grade corresponding to a row.

| Overall Grade Predictions |      |      |      |      |      |       |       |
|---------------------------|------|------|------|------|------|-------|-------|
| Based on Flipped Trends   |      |      |      |      |      |       |       |
|                           | A    | B    | C    | D    | F    | $x_1$ | $x_2$ |
| A                         | 0.73 | 0.05 | 0.00 | 0.00 | 0.00 | 4     | 3.6   |
| B                         | 0.27 | 0.83 | 0.23 | 0.00 | 0.00 | 14    | 14.5  |
| C                         | 0.00 | 0.12 | 0.61 | 0.25 | 0.10 | 8     | 8.4   |
| D                         | 0.00 | 0.00 | 0.16 | 0.55 | 0.10 | 5     | 4.6   |
| F                         | 0.00 | 0.00 | 0.00 | 0.20 | 0.80 | 5     | 5.0   |

$x_1$  = Cumulative grade distribution following test three

$x_2$  = Predicted final grade distribution for course

| Based on Conventional Trends |     |     |     |     |     |       |       |
|------------------------------|-----|-----|-----|-----|-----|-------|-------|
|                              | A   | B   | C   | D   | F   | $x_1$ | $x_2$ |
| A                            | 0.7 | 0.1 | 0.0 | 0.0 | 0.0 | 4     | 4.6   |
| B                            | 0.4 | 0.5 | 0.0 | 0.1 | 0.0 | 14    | 8.9   |
| C                            | 0.0 | 0.3 | 0.7 | 0.1 | 0.0 | 8     | 10.3  |
| D                            | 0.0 | 0.1 | 0.3 | 0.6 | 0.3 | 5     | 6.8   |
| F                            | 0.0 | 0.0 | 0.0 | 0.3 | 0.6 | 5     | 4.3   |

| Final Exam Scores                  |      |      |      |      |      |       |       |
|------------------------------------|------|------|------|------|------|-------|-------|
| Prediction Based on Flipped Trends |      |      |      |      |      |       |       |
|                                    | A    | B    | C    | D    | F    | $y_1$ | $y_2$ |
| A                                  | 0.53 | 0.17 | 0.15 | 0.00 | 0.05 | 7     | 6.5   |
| B                                  | 0.28 | 0.37 | 0.35 | 0.00 | 0.16 | 9     | 8.8   |
| C                                  | 0.10 | 0.32 | 0.25 | 0.25 | 0.09 | 5     | 6.9   |
| D                                  | 0.00 | 0.09 | 0.00 | 0.25 | 0.19 | 4     | 3.9   |
| F                                  | 0.08 | 0.05 | 0.25 | 0.50 | 0.51 | 11    | 9.9   |

| Prediction Based on Conventional Trends |      |      |      |      |      |       |       |
|---|------|------|------|------|------|-------|-------|
|   | A    | B    | C    | D    | F    | $y_1$ | $y_2$ |
| A                                       | 0.65 | 0.10 | 0.00 | 0.06 | 0.00 | 7     | 5.7   |
| B                                       | 0.15 | 0.33 | 0.13 | 0.00 | 0.00 | 9     | 4.7   |
| C                                       | 0.20 | 0.32 | 0.26 | 0.18 | 0.00 | 5     | 6.3   |
| D                                       | 0.00 | 0.17 | 0.46 | 0.35 | 0.38 | 4     | 9.4   |
| F                                       | 0.00 | 0.08 | 0.14 | 0.42 | 0.63 | 11    | 10.0  |

$y_1$  = Grade distribution of test #3

$y_2$  = Predicted grade distribution of final exam

## Results

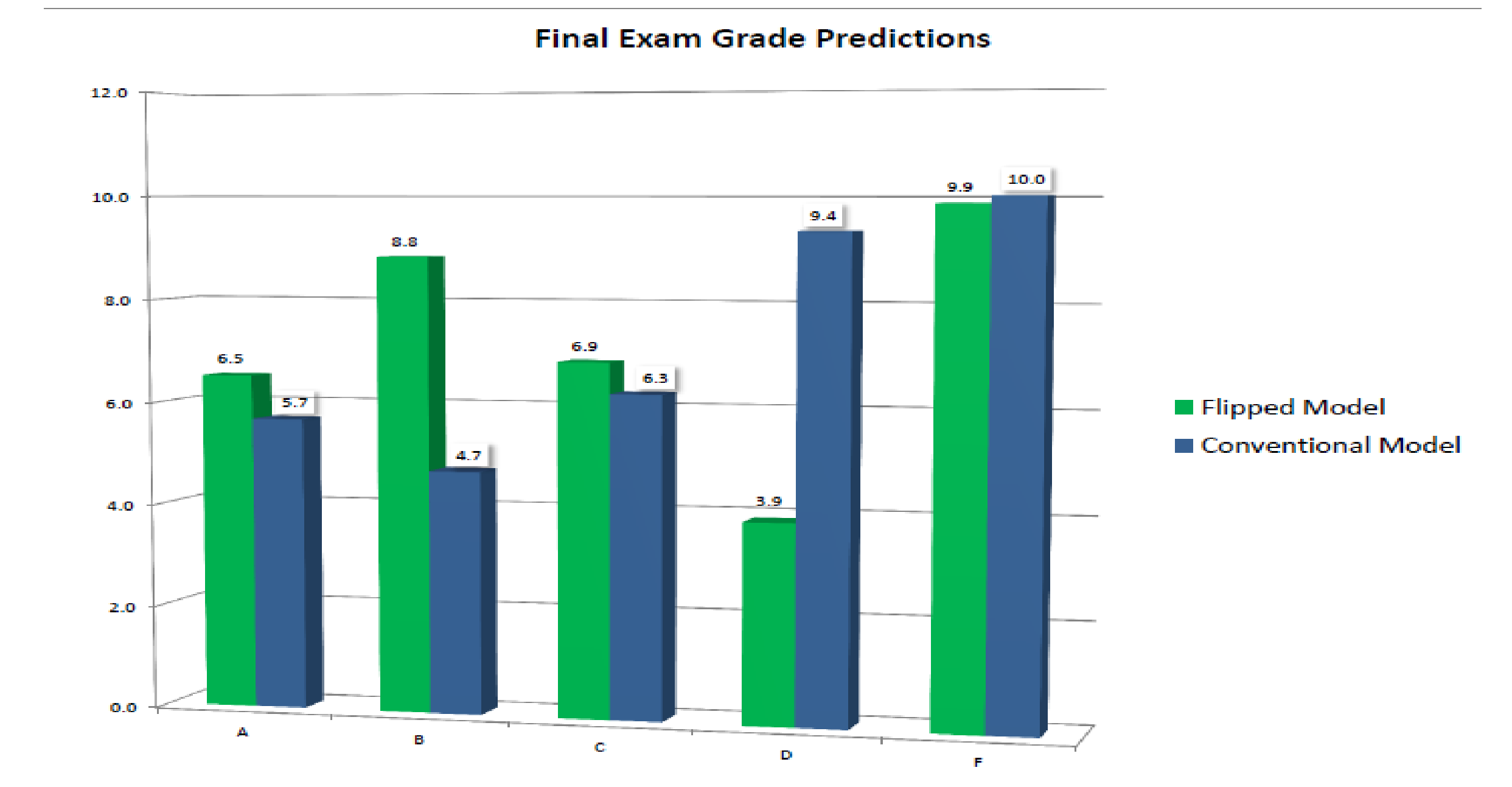
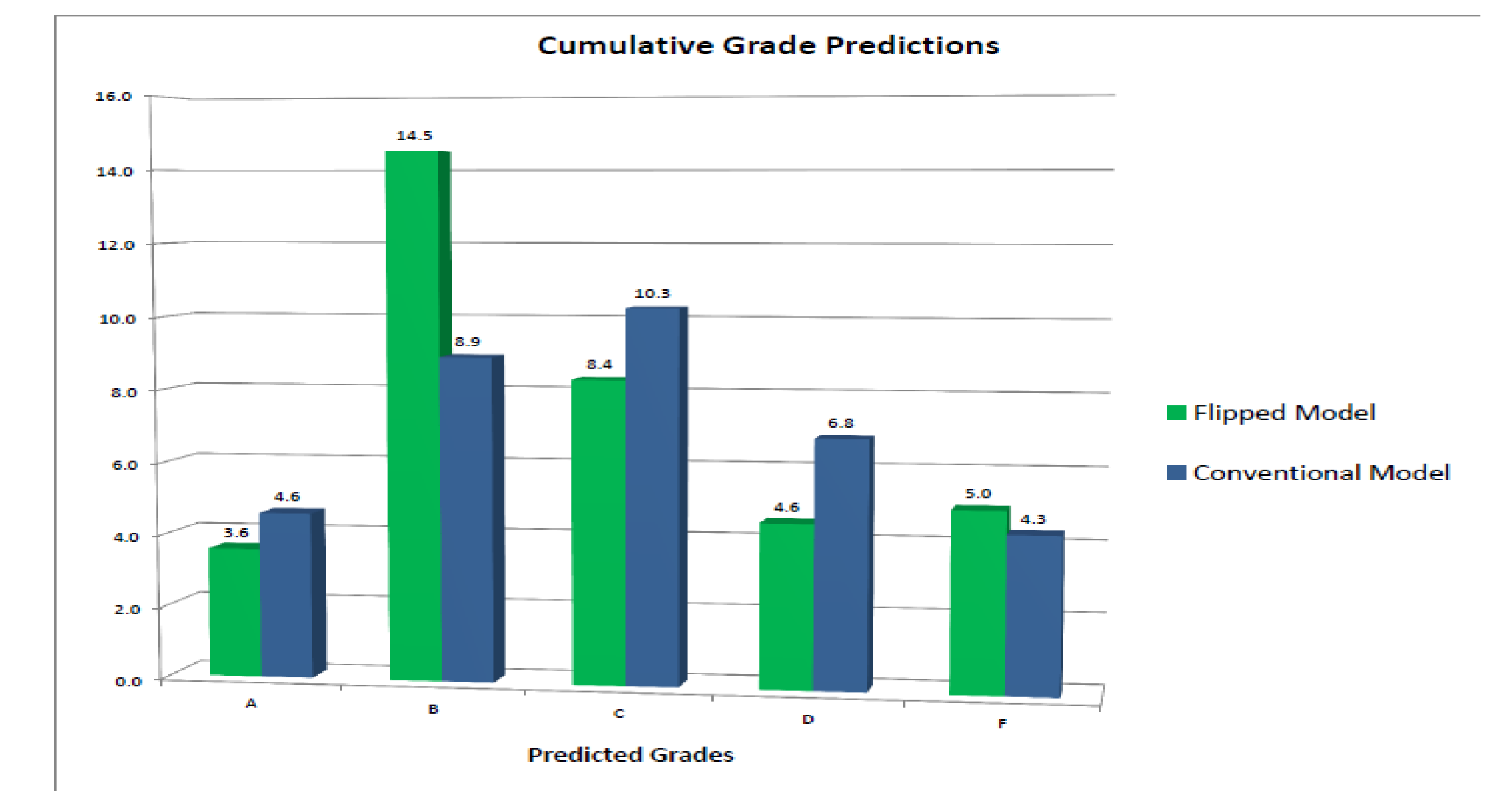
Resulting predictions based on flipped grade trends

- Mode of B
- Average grade of 2.2

Resulting predictions based on conventional grade trends

- Modes of C & D
- Average grade of 2.01

## Conclusion



The data suggests that a student enrolled in a flipped class will tend to score higher on average than in a conventional class.

## Addendums

With our limited amount of data we were not able to create a conclusive statement about the distribution of grades in the flipped classroom.

It should also be noted that the withdrawals were counted as fails, which contributed to the high number of F's for both classes.

## Acknowledgements

Lay, David C. *Linear Algebra and Its Applications*. 4<sup>th</sup> ed. Page 253. Boston: Pearson Education, Inc, 2012. Print.

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