California State University, Long Beach,
Department Of Mathematics and Statistics

Game Theory: Applications in Linear Algebra

Math 247 Linear Algebra
December 2, 2013
Fall 2013

By: Julia, Morgan, Lori, Lisa, & Chris

Introduction

Game theory is used in a situation (or "game") when the choice made by one player affects the outcome of both players. It is a way to use matrices to calculate the positive or negative outcomes for two "players" depending on their choices. This allows each player to make an educated choice.

Method

Nash Equilibrium:

- Provides a way to predict what will happen if several people or several institutions are making decisions simultaneously
- Maximizing payoff (gain) while trying to minimize the other participant's payoff (loss)
- When making a decision it enables Player A to take into account Player B's decision
- Denoted in a matrix format
- Applied in many different sub-categories such as Zero Summation, Reduction by Dominance, etc.

Reduction By Dominance:

1. Check whether there is any row in the matrix that is dominated by another row ($r_i \leq r_j$). If there is one, delete it
2. Check whether there is any column in the matrix that is dominated by another column ($c_i \geq c_j$). If there is one, delete it.
3. Repeat steps 1 and 2 until there are no dominated rows or columns.

i. Dominance:

- Each entry of the row is greater (or less) than the respective entries of another row.
- Begin with the first row (numerical order)

Examples

Method I: A Game of Chance

Suppose we have two players, A and B, and they are playing 4 basketballs games within their home region. The probability that A or B wins any particular game is ½. We can denote a win as 1, and a loss as 0, displayed on a 2*2 matrix to display how many games player A and player B won. The player with the most wins out of the 4 basketball games, is declared the winner. A program can easily be written in MatLab to illustrate this scenario.

Matlab Code

```
%P: probability P
%S: size of the matrices A and B
%A: A(i,j)=1 <=> (i,j) & (i,j+1) are connected
%B: B(i,j)=1 <=> (i,j) & (i+1,j) are connected
function [A,B]=createMatrix(P,S)
A = zeros(S);
B = zeros(S);
 for i = 1:S
   for j = 1:S
      r = rand(1);
      if r>P
         A(i,j) = 0;
      elseif r<P
         A(i,j) = 1;
      end
   end
end
 for i = 1:S
   for j = 1:S
      r = rand(1);
      if r>P
         B(i,j) = 0;
      elseif r<P
         B(i,j) = 1;
      end
   end
end
```

Method II: Game Theory in Football

Scenario: You are the coach of Team A that is currently on the offense while Team B is on the defense. You have five play strategies but are unsure of which to select. You also know that Team B has three defensive play strategies that they could choose from.

Over the years, Teams have recorded the average yardage gained by team A for each combination of strategies organized in this 5x3 matrix:

$$\begin{array}{cc} & \begin{array}{ccc} & B & \\ 1 & 2 & 3 \end{array} \\ A \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & -1 & 5 \\ 7 & 5 & 10 \\ 15 & -4 & -5 \\ 5 & 0 & 10 \\ -5 & -10 & 10 \end{bmatrix} \end{array} \rightarrow \begin{bmatrix} \cancel{0} & \cancel{-1} & \cancel{5} \\ 7 & 5 & 10 \\ 15 & -4 & -5 \\ \cancel{5} & \cancel{0} & \cancel{10} \\ \cancel{-5} & \cancel{-10} & \cancel{10} \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 5 & 10 \\ 15 & -4 & -5 \end{bmatrix}$$

The table is actually a payoff matrix. We reduce by dominance. Row 2 dominates rows 1, 4, and 5 since the payoffs are larger no matter what play the other team chooses. Thus we can eliminate all three of those rows:

Looking at the columns now, we want to now figure out the most likely play Team B is going to make, or the lowest payoff for Team A. Since Column 2 dominated Column 1 in this case, we get rid of the first column:

We switch back to the rows now and again looking for the highest payoff. Since Row 2 dominates Row 3 we'll get rid of the third row:

$$\begin{bmatrix} 5 & 10 \\ -4 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & 10 \end{bmatrix}$$

Since we're left with one row we switch back to columns now along with the smallest payoff. Column 2 dominates Column 3 in this case and we're left with a 1x1 matrix that gives the best possible play:

$$[5]$$

Results

This example illustrates Game Theory through the reduction by dominance method. By having two players trying to make the best possible choice to have a maximum playoff while minimizing the opponents payoff. As team A is taking account team's B counter decision from their own decision, the final result becomes to use play 2,2

Where can I use Game Theory?

Game theory is useful anywhere where there are two or more participants, each with two or more choices. This includes economics, political strategy, poker games, sports, making personal decisions, and more!

Summary

Through the integration of linear algebra with game theory, matrices can be incorporated to help organize information about the choices of multiple competitors. By using the Nash Equilibrium and linear algebra, the competitors can make informed quantitative decisions through the process of matrix reduction.

Conclusion

Game theory is a useful way to keep track of choices and calculate outcomes. It can help you figure out what an opponent might do, and help you make an informed choice based on the information you have.

Acknowledgements

- Chapter G: Game Theory (pg 5) http://www.zweigmedia.com/pdfs/GameTheory.pdf
- http://math.ucr.edu/home/baez/games/games_12.html
- http://en.wikipedia.org/wiki/Nash_equilibrium