Linear Algebra Applications to Kirchhoff Laws for Adaptive Circuit Analysis

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#: Introduction

Linear Algebra Can Help Me Master Electronics?

Yes! The applications of linear algebra are endless, especially for engineers. In electronics, engineers are responsible for interfacing new components with old systems, repairing systems, and so on. This involves a multitude of unknown currents which are need to be accounted for since every resistor and capacitor have operating threshold values in which they operate. In other words, if you don't account for the change in current, you can cause a minor explosion.

Linear algebra can help by turning Kirchoff Node and Voltage equations into a manage Matrix whose homogenous solutions set is composed of the desired currents.

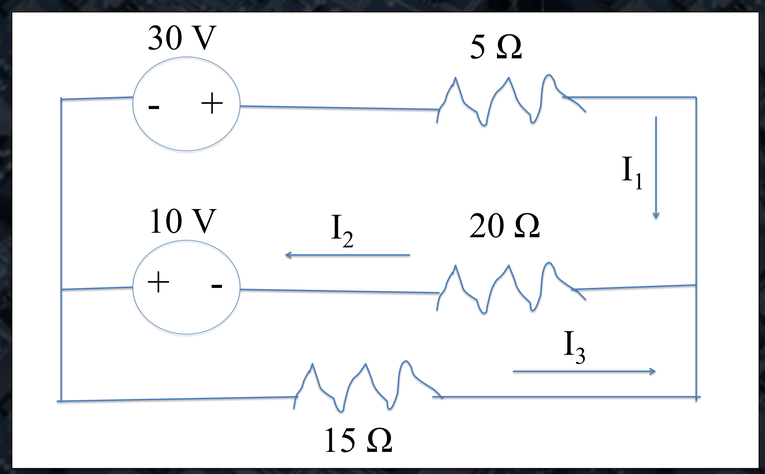
Kirchhoff Node and Voltage Equations

Kirchhoff was the physicist responsible for proving that energy is conserved in a circuit. This means that current entering a node, a point in the circuit where one or more components converge, equals the current leveling a node. Likewise, Kirchhoff's voltage law sattes that through any complete path through the circuit, the voltage gained equals the voltage lost.

To demonstrate how Linear algebra can be utilized to simplify circuit analysis, a simple circuit, containing 3 resistors and 2 power sources is examined.

#: Methods

The circuit used for this project is illustrated below



The circuit has two sources of voltage, a 10 V and 30 V. There are three resistors in parallel of 5, 20, and 15 Ohms. The system of linear equations that can represent the circuit are as followed:

$$I_1 + I_3 - I_2 = 0$$
$$5I_1 + 20I_2 = 40 \text{ V}$$
$$5I_1 + 15I_2 = 20 \text{ V}$$

#: Methods

The code, taking in voltage and resistance, that was designed for this specific circuit is as followed

```
public void swapRows(int row1,int row2)
    double temp;
    for(int j=0;j<numEq;j++){   // SWAPPING CO-EFFICIENT ROWS
      temp=m[row1][j];
      m[row1][j]=m[row2][j];
      m[row2][j]=temp;
    temp=constants[row1];   // SWAPPING CONSTANTS VECTOR
    constants[row1]=constants[row2];
    constants[row2]=temp;
  }
public void eliminate()
  {
    int i,j,k,l;
    for(i=0;i<numEq;i++){   // i -> ROW ; MATRIX ORDER
DECREASES DURING ELIMINATION
      // FIND LARGEST CO-EFFICIENTSOF THE CURRENT COLUMN
MOVING ROW-WISE
      double largest=Math.abs(m[i][i]);
      int index=i;
      for(j=i+1;j<numEq;j++){
        if(Math.abs(m[j][i])>largest){
          largest=m[j][i];
          index=j;
        }
      swapRows(i,index);  // SWAPPING i-th ROW to index-th ROW
      for(k=i+1;k<numEq;k++){
        double factor=m[k][i]/m[i][i];
        // PROCESSING COLUMN WISE
        for(l=i;l<numEq;l++){
          m[k][l]-=factor*m[i][l];
      constants[k]-=factor*constants[i];  // PROCESSING CONSTANTS

public void solve()
    for(int i=numEq-1;i>=0;i--){
      solution[i]=constants[i];   // COPY
      for(int j=numEq-1;j>i;j--){
        solution[i]-=m[i][j]*solution[j];
      solution[i]/=m[i][i];
```

#: Results_&_Summary

All currents are calculated quickly and accurately using row operations to solve the augmented matrix

Our code allows for votage and resistance input and outputs currents through each resistor. While this code is

```
Enter the Number of resistor values : 3
Enter the resistance(s) and voltages (respectively) for equation 1 :
Enter Resistance 1 : 5
Enter Resistance 2 : 20
Enter Resistance 3 : 0
Enter Voltage for this equation : 40
Enter the resistance(s) and voltages (respectively) for equation 2 :
Enter Resistance 1 : -5
Enter Resistance 2 : 20
Enter Resistance 3 : 15
Enter Voltage for this equation : 10
Enter the resistance(s) and voltages (respectively) for equation 3 :
Enter Resistance 1 : 5
Enter Resistance 2 : 0
Enter Resistance 3 : 15
Enter Voltage for this equation : 30
You Have Entered The Following Equations for the Circuit :
 +5.0A  +20.0B  +0.0C  = 40.0
 -5.0A  +20.0B  +15.0C  = 10.0
 +5.0A  +0.0B  +15.0C  = 30.0

 Current for each equation is :
A = 4.0
B = 1.0
C = 0.6666666666666666
```

specific for the circuit we are working with, this can be used as a template for more complicated circuit designs. The Node and Voltage Laws still apply to more complicated systems because of the conservation of energy in the systme. More complicated circuits will merely have more equations and more variables.

#: Conclusion

Components have threshold voltage and currents that they operate under

Resistors and capacitors will explode or catch fire if the current and/or voltage is too high. Using this code as a launch pad, engineers can quickly calculate the current through each component and verify that it is within the operating threshold.

Linear Algebra eases the process of interfacing and repair components

Since current through each component can be monitored using a simple augmented matrix, engineers can focus more on their hardware and less on the math.

#: Acknowledgements

Lay, David C. *Linear Algebra and Its Applications.* 4th Ed. Pearson Education, 2012.

Rizzoni, Giorgio. *Principles and Applications of Electrical Engineering.* 5th Ed. McGraw-Hill, 2007.