Application of Linear Algebra in Macroeconomics

By: Bryant Lopez, Eriton Sena, & Raymond Cheung

What is Macroeconomics?

- The study of the workings of the economy as a whole.
- Government policies in accordance with producer's and consumer's behaviors.

Focus

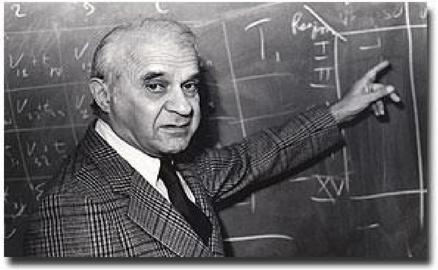
- Using the matrix equation Ax=b to explain the behavior of producers given a consumer demand.
- Encapsulating the explanation under an assumed open economy.

Input-Output Model

- Divides all sectors of an economy into a table for data analysis of the interdependent relations between the sectors.

Leontief's Developed Model

- Wassily Leontief, a Russian-American economist, utilized linear algebra and macroeconomic theory in a combinatorial setting by effectively developing a new model under the input-output analysis.

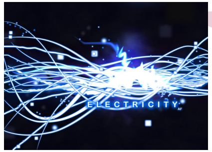


Method of Leontief's Input-Output Model

- Step 1: Formulate input-output table.
- Step 2: Use table values to construct a consumption matrix (C) where every value in C is equal to its sector's total input value divided by its respective gross output.
- Step 3: Calculate desired production vector using the open sector's demand (d).

Simple's Economy

E	-	Electricity
T	-	Transportation
L	-	Labor
O.S.	-	Open Sector



Input-Output Table

Numbers represented in Billions of Dollars	E	T	L	O.S.
E	0.50	3.20	6.80	9.50
T	3.00	1.00	4.00	8.00
L	1.20	5.00	1.20	10.6
Total Gross Output	20.0	18.0	16.0	

- C Is always an nxn matrix because Leontief's model always evaluates an equal amount of sectors.

$$C = \begin{bmatrix} 0.025 & 0.178 & 0.425 \\ 0.150 & 0.056 & 0.250 \\ 0.06 & 0.278 & 0.067 \end{bmatrix}$$

- d represents exogenous (outside of the model) values of demand from the open sector.

$$d = \begin{bmatrix} 9.5 \text{ billion} \\ 8.0 \text{ billion} \\ 10.6 \text{ billion} \end{bmatrix}$$

Calculations

P = CP + d → (I – C)P = d → P = [(I-C)^-1]d
Where P is the value of the production vector.

$$(I-C) = \begin{bmatrix} 0.975 & -0.178 & -0.425 \\ -0.150 & 0.944 & -0.250 \\ -0.060 & -0.278 & 0.933 \end{bmatrix}$$

$$(I-C)^{-1} = \begin{bmatrix} 1.124 & 0.394 & 0.618 \\ 0.215 & 1.225 & 0.426 \\ 0.136 & 0.390 & 1.239 \end{bmatrix}$$

$$(I-C)^{-1}d = \begin{bmatrix} 1.124 & 0.394 & 0.618 \\ 0.215 & 1.225 & 0.426 \\ 0.136 & 0.390 & 1.239 \end{bmatrix} \begin{bmatrix} 9.5 \text{ billion} \\ 8.0 \text{ billion} \\ 10.6 \text{ billion} \end{bmatrix}$$

Results

$$P = \begin{bmatrix} 20.381 \text{ billion} \\ 16.358 \text{ billion} \\ 17.545 \text{ billion} \end{bmatrix}$$

- This production vector indicates the amount 'Simple' needs to produce in order to satisfy its intermediate demand and final open sector demand.

Summary & Conclusion

- Leontief's Input-Output Model can be used to find the monetary output each sector in 'Simple' would need to produce in order to satisfy their economy. The application of Leontief's Model expands further on sector-to-sector analysis, including economic forecasting and regression analysis.

Acknowledgements

Lay, David. *Linear Algebra and its Applications*. Third. MA: Pearson Education, Inc., 2006. Print.

http://www.unc.edu/~marzuola/Math547_S13/Math547_S13_Projects/J_Zu_Section001_InputOutput.pdf