LEONTIEF INPUT-OUTPUT MODEL

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LEONTIF MODEL PURPOSE

- Ability to see how change in demand for one industry affects economy
- Understand and manipulate economy of country or region
- Estimate total production needed to satisfy an economy
- Often used for city planning or analyzing national economy

Questions

- How much external demand can we meet?
- How much must we produce to meet certain demand?

EXAMPLE

Find the production level of each of the three industries in the period of one week to satisfy the internal demands.

To Produce $1 of coal

Industry resource	Input/ cost
Coal	$0.30
Electricity	$0.30
Automobile	$0.30

To Produce $1 Electricity

Industry resource	Input/ cost
Coal	$0.40
Electricity	$0.10
Automobile	$0.50

To Produce $1 of Automobile

Industry resource	Input/ Cost
Coal	$0.30
Electricity	$0.50
Automobile	$0.20

LEONTIF CLOSED MODEL

System of equations: The total consumption must be equal to the total production of all the industries.

$$0.30 * coal \ \ + 0.40 + electricity + 0.30 * auto = coal\ production$$

$$0.30 \ * coal + 0.10 \ * electricity + 0.50 \ * auto = electricity\ production$$

$$0.30 \ * coal \ \ + 0.50 \ * electricity + 0.20 \ * auto = automobile\ production$$

LEONTIF CLOSED MODEL

The Input Output Matrix

$$A = \begin{bmatrix} 0.3 & 0.3 & 0.3 \\ 0.4 & 0.1 & 0.5 \\ 0.3 & 0.6 & 0.2 \end{bmatrix}$$

LEONTIF CLOSED MODEL

We are then looking for a vector *P* satisfying *AP=P* and with nonnegative components, at least one of which is positive.

Using I, the Identity matrix.

$$AP = P \Leftrightarrow AP*I = P*I \Leftrightarrow AP*I - P*I = 0 \Leftrightarrow$$

$$(A\text{-}I)P = 0$$

LEONTIF CLOSED MODEL

(A-I)P = 0 .

The augmented (A-I)P would be:

$$(A-I)P = \left[\begin{array}{ccc|c} 0.3-1 & 0.3 & 0.3 & 0 \\ 0.4 & 0.1-1 & 0.5 & 0 \\ 0.3 & 0.6 & 0.2-1 & 0 \end{array}\right]$$

Which results to:

$$\begin{bmatrix} -0.7 & 0.3 & 0.3 & 0 \\ 0.4 & -0.9 & 0.5 & 0 \\ 0.3 & 0.6 & -0.8 & 0 \end{bmatrix}$$

LEONTIF CLOSED MODEL

The reduced row echelon form of this equation results to:

$$\begin{bmatrix} 1 & 0 & -0.82 & 0 \\ 0 & 1 & -0.92 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

LEONTIF CLOSED MODEL

To determine the amount needed for each industry to produce, set it to parametric form.

$$\begin{cases} p_1 = 0.82t \\ p_2 = 0.92t \\ p_3 = t \end{cases}$$

LEONTIF CLOSED MODEL – RESULT ANALYSIS

If we multiply the values by setting t to 100, the solution would be easy to read.

To maintain the economic system efficiently, we would need…

- $82.00 worth of coal
- $92.00 worth of electricity
- $100 of the automobiles

$$\begin{cases} p_1 = 82\,units \\ p_2 = 92\,units \\ p_3 = 100\,units \end{cases}$$

LEONTIF'S OPEN MODEL

-It is simply not common to find a closed economy. In most real-world applications there must be the inclusion of outside demand.

-Therefore we have $p_1 = m_{i1}p_1 + m_{i2}p_2 + \ldots + m_{in}pn + d_i$

where d_i is the demand from the i^{th} outside industry and the rest as in the closed model.

LEONTIF'S OPEN MODEL (CONT.)

-With this we are given the following linear system:

$$P = AP + d$$

where $d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$ is the demand vector. Thus it can be easily solved

$$P = AP+d \Leftrightarrow (I\text{-}A)P = d \Leftrightarrow P = (I\text{-}A)^{-1}d$$

provided that the matrix (I-A) be invertible.

CONCLUSION

- Powerful economic analysis tool
- Application is important to maximize output and efficiency between industries.
- Predicting and making projections for the future
- Analysis of national economy