

# LEONTIEF INPUT-OUTPUT MODEL

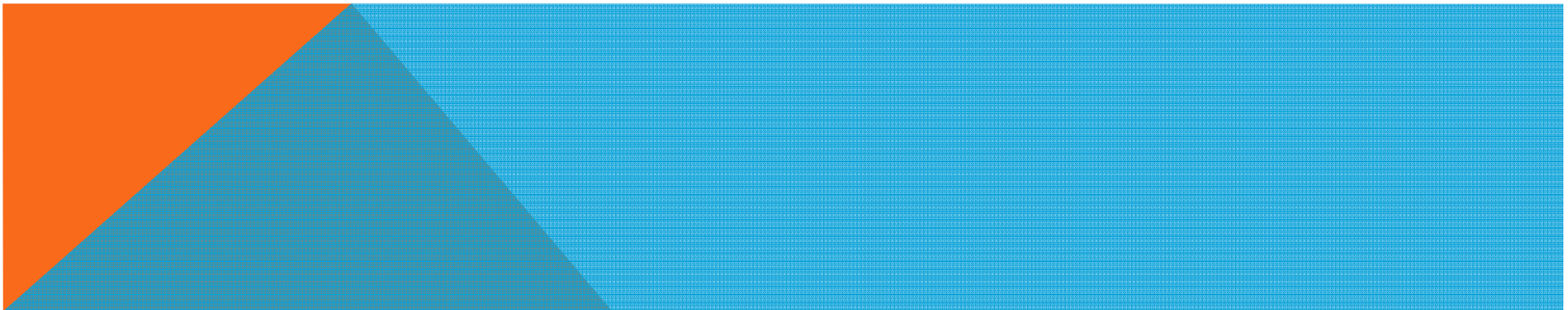
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# LEONTIF MODEL PURPOSE

- Ability to see how change in demand for one industry affects economy
- Understand and manipulate economy of country or region
- Estimate total production needed to satisfy an economy
- Often used for city planning or analyzing national economy

## Questions

- How much external demand can we meet?
- How much must we produce to meet certain demand?



# EXAMPLE

Find the production level of each of the three industries in the period of one week to satisfy the internal demands.



To Produce \$1 of coal

<u>Industry</u> <u>resource</u>	<u>Input/</u> <u>cost</u>
Coal	\$0.30
Electricity	\$0.30
Automobile	\$0.30



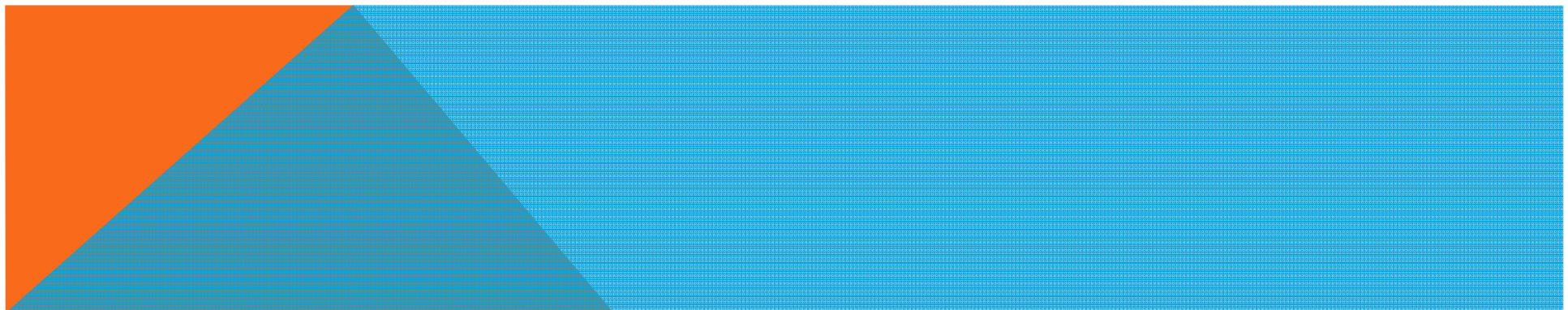
To Produce \$1 Electricity

<u>Industry</u> <u>resource</u>	<u>Input/</u> <u>cost</u>
Coal	\$0.40
Electricity	\$0.10
Automobile	\$0.50



To Produce \$1 of Automobile

<u>Industry</u> <u>resource</u>	<u>Input/</u> <u>Cost</u>
Coal	\$0.30
Electricity	\$0.50
Automobile	\$0.20



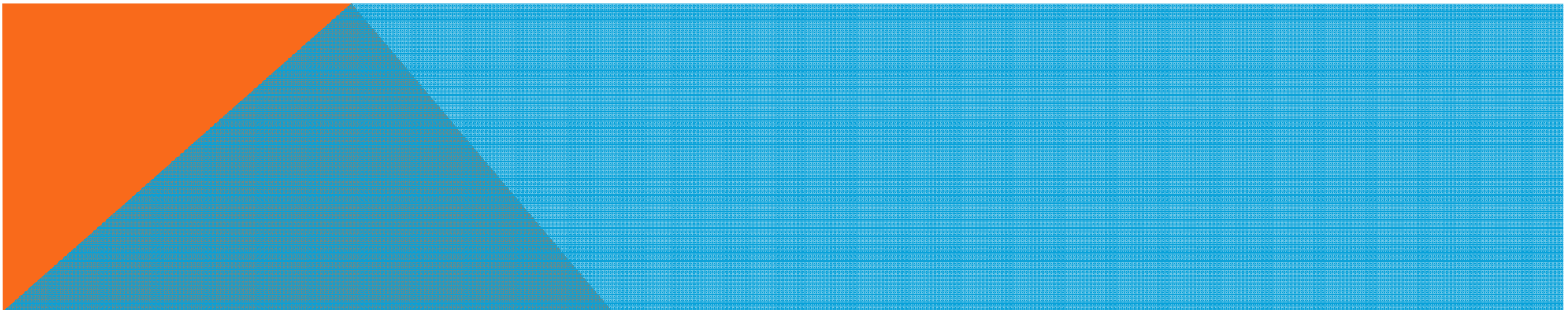
# LEONTIF CLOSED MODEL

System of equations: The total consumption must be equal to the total production of all the industries.

$$0.30 * coal + 0.40 * electricity + 0.30 * auto = coal\ production$$

$$0.30 * coal + 0.10 * electricity + 0.50 * auto = electricity\ production$$

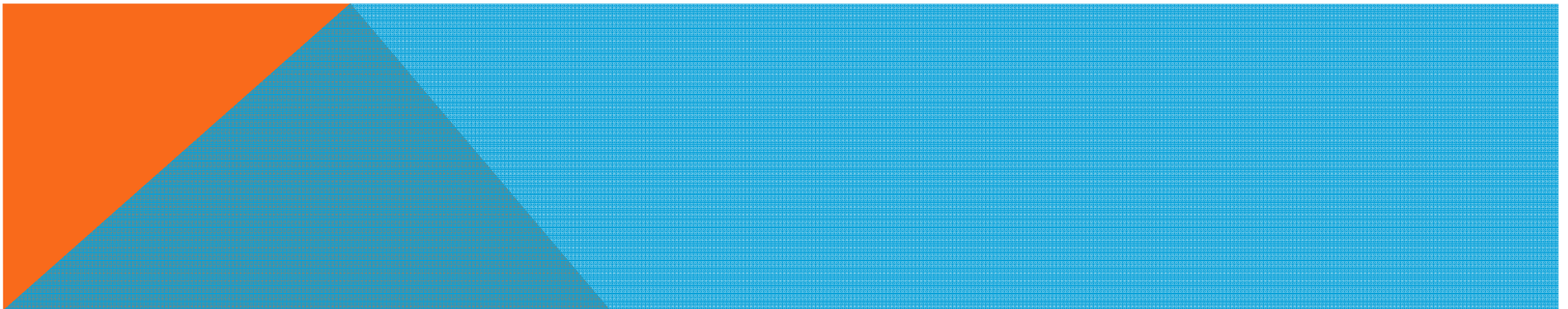
$$0.30 * coal + 0.50 * electricity + 0.20 * auto = automobile\ production$$



# LEONTIF CLOSED MODEL

The Input Output Matrix

$$A = \begin{bmatrix} 0.3 & 0.3 & 0.3 \\ 0.4 & 0.1 & 0.5 \\ 0.3 & 0.6 & 0.2 \end{bmatrix}$$





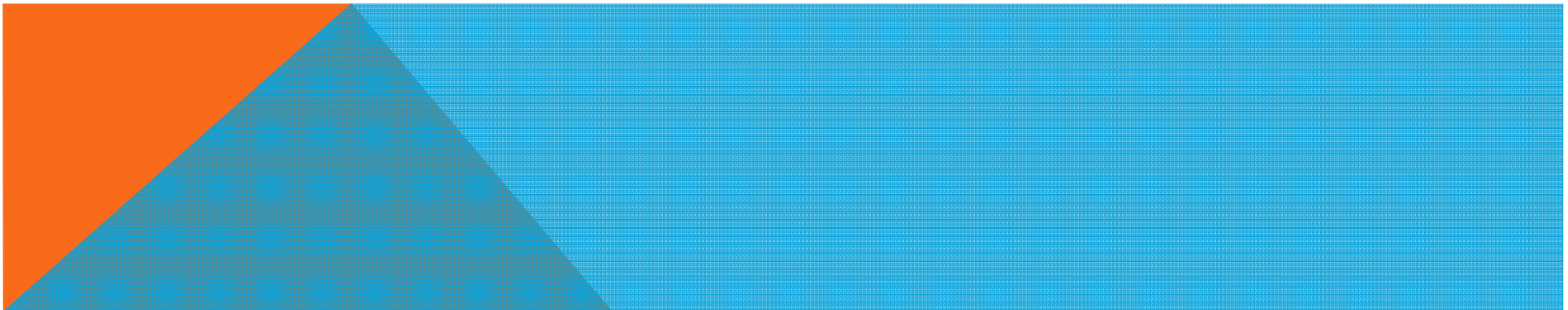
# LEONTIF CLOSED MODEL

We are then looking for a vector  $P$  satisfying  $AP=P$  and with nonnegative components, at least one of which is positive.

Using  $I$ , the Identity matrix.

$$AP = P \Leftrightarrow AP^*I = P^*I \Leftrightarrow AP^*I - P^*I = 0 \Leftrightarrow$$

$$(A-I)P = 0$$



# LEONTIF CLOSED MODEL

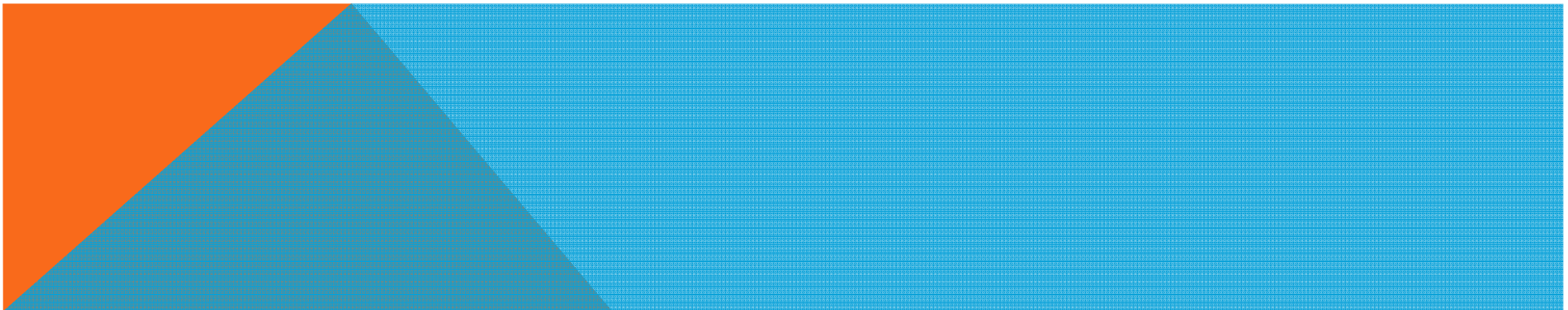
$$(A-I)P = 0.$$

The augmented  $(A-I)P$  would be:

$$(A-I)P = \left[ \begin{array}{ccc|c} 0.3 - 1 & 0.3 & 0.3 & 0 \\ 0.4 & 0.1 - 1 & 0.5 & 0 \\ 0.3 & 0.6 & 0.2 - 1 & 0 \end{array} \right]$$

Which results to:

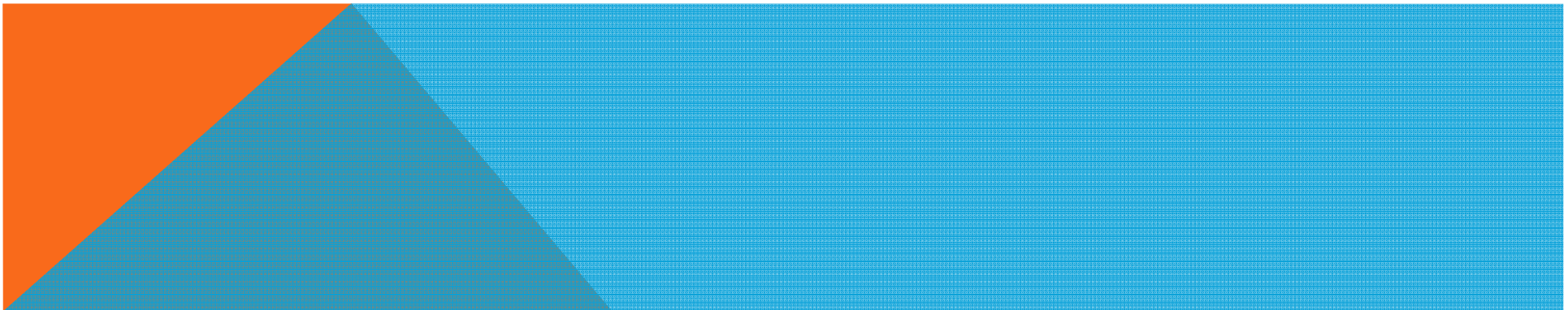
$$\left[ \begin{array}{cccc} -0.7 & 0.3 & 0.3 & 0 \\ 0.4 & -0.9 & 0.5 & 0 \\ 0.3 & 0.6 & -0.8 & 0 \end{array} \right]$$



# LEONTIF CLOSED MODEL

The reduced row echelon form of this equation results to:

$$\begin{bmatrix} 1 & 0 & -0.82 & 0 \\ 0 & 1 & -0.92 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$





# LEONTIF CLOSED MODEL

To determine the amount needed for each industry to produce, set it to parametric form.

$$\begin{cases} p_1 = 0.82t \\ p_2 = 0.92t \\ p_3 = t \end{cases}$$



# LEONTIF CLOSED MODEL – RESULT ANALYSIS

If we multiply the values by setting  $t$  to 100, the solution would be easy to read.

To maintain the economic system efficiently, we would need...

- \$82.00 worth of coal
  - \$92.00 worth of electricity
  - \$100 of the automobiles
- $$\begin{cases} p_1 = 82 \text{ units} \\ p_2 = 92 \text{ units} \\ p_3 = 100 \text{ units} \end{cases}$$



# LEONTIF'S OPEN MODEL

-It is simply not common to find a closed economy. In most real-world applications there must be the inclusion of outside demand.

-Therefore we have  $p_1 = m_{i1}p_1 + m_{i2}p_2 + \dots + m_{in}pn + d_i$

where  $d_i$  is the demand from the  $i^{\text{th}}$  outside industry and the rest as in the closed model.



## LEONTIF'S OPEN MODEL (CONT.)

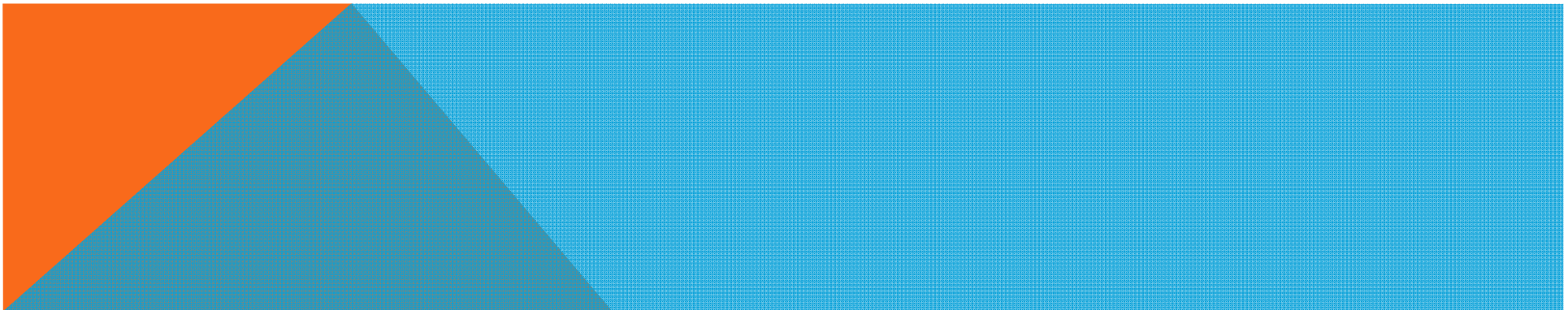
-With this we are given the following linear system:

$$P = AP + d$$

where  $d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$  is the demand vector. Thus it can be easily solved

$$P = AP + d \Leftrightarrow (I - A)P = d \Leftrightarrow P = (I - A)^{-1}d$$

provided that the matrix  $(I - A)$  be invertible.



# CONCLUSION

- Powerful economic analysis tool
- Application is important to maximize output and efficiency between industries.
- Predicting and making projections for the future
- Analysis of national economy

