SMOOTHING

Maximizing probability

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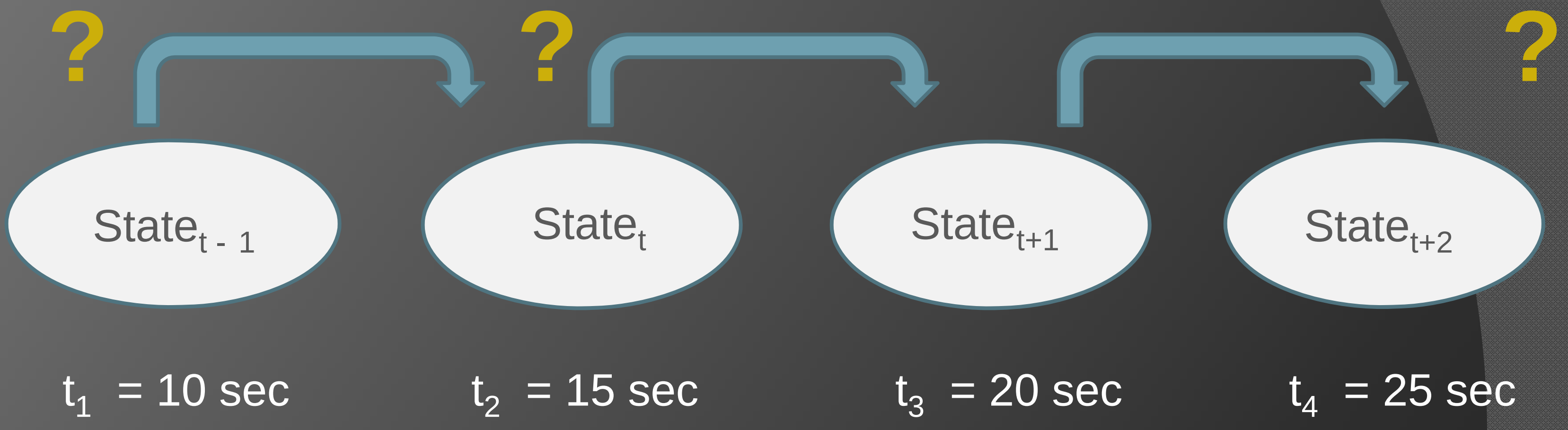
The Transition Model



Three possible inference tasks

Filtering – Calculating the most likely state we are in

Prediction – Estimating the probability of a state at a future time

Smoothing – Estimating the probability of a past state

Whack-a-Mole

We want to guess which pie slice a mole will appear in

If I'm in state 1, what's the probability I go to state 2?

To

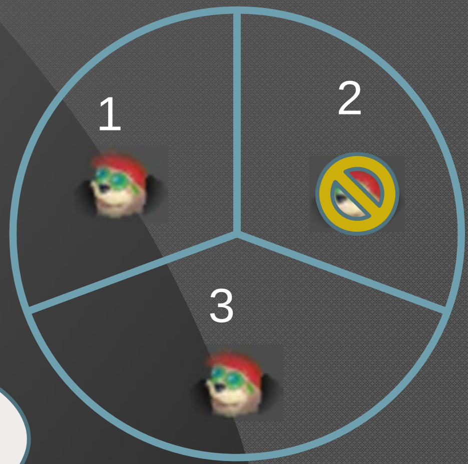
From

$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & M = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}
\end{array}
$$

The Transition Matrix

Filtering – At Time = 1

Furthermore, we only see the mole with a 75% accuracy



Mole Starts in 1

60% in 3
40% in 2

t_1 = 5 sec

t_2 = 10 sec

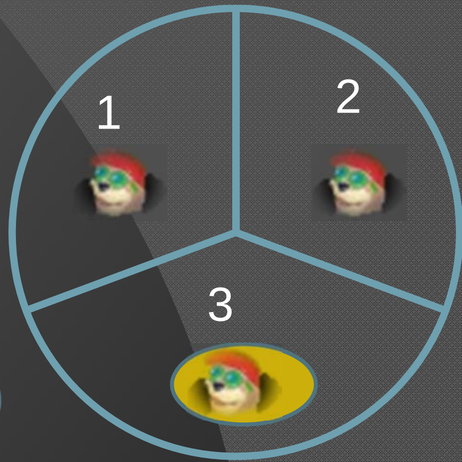
t_3 = 15 sec

Measured that the mole was NOT in slice 2 at time one

$$[\ 1\ 0\ 0\ ] \times \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \times [\ 3/4\ 1/4\ 3/4\ ]$$

$$= [\ 0\ .40\ .60\ ]$$

Filtering – At Time = 2



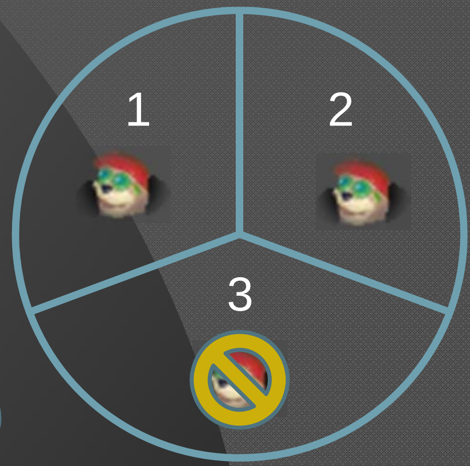
Mole Starts in 1

t_1 = 5 sec	t_2 = 10 sec	t_3 = 15 sec
60% in 3 40% in 2	52% in 3 34% in 1	

$$[\ 0\ \ .40\ \ .60\ ] \times \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \times [\ 1/4\ \ 1/4\ \ 3/4\ ]$$

$$= [\ .3478\ \ .1304\ \ .5217\ ]$$

Filtering – At Time = 3



Mole Starts in 1

60% in 3
40% in 2

t_1 = 5 sec

52% in 3
34% in 1

t_2 = 10 sec

46% in 2
45% in 1

t_3 = 15 sec

$$[\ .3478\ \ .1304\ \ .5217\ ] \times \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} \times [\ 3/4\ \ 3/4\ \ 1/4\ ]$$

$$=[\ .4525\ \ .4693\ \ .0782\ ]$$

Smoothing – Estimating State 3

Furthermore, we only see the mole with a 75% accuracy

46% in 2
45% in 1

t_1 = 5 sec

t_2 = 10 sec

t_3 = 15 sec

$$[\ 1\ \ 1\ \ 1\ ] \times [\ 3/4\ \ 3/4\ \ 1/4\ ] \times \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \\ 2/3 & 1/3 & 0 \end{bmatrix}^T$$

$$=[.3333\ \ .2380\ \ .4286]$$

Smoothing – Estimating State 2

Furthermore, we only see the mole with a 75% accuracy

46% in 2
45% in 1

t_1 = 5 sec

t_2 = 10 sec

t_3 = 15 sec

Combining **Filtering(forward)** and **Smoothing(backward)** estimates, we can get **a more accurate** estimate of where the mole is.

Filtering Row Vector ✖ Smoothing Row Vector ✖ Transition Matrix

=[.3128 .0838 .6033]

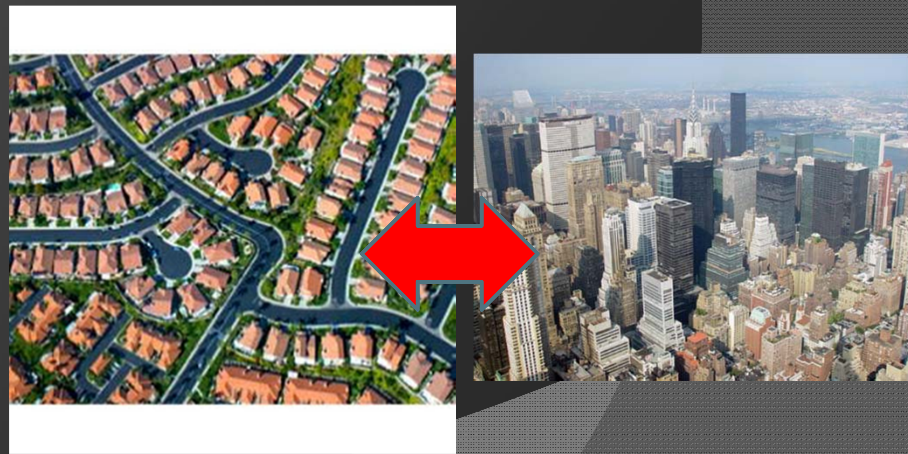
Filtering & Smoothing: **60%**
Filtering: **52%**

Applications of Filtering and Smoothing

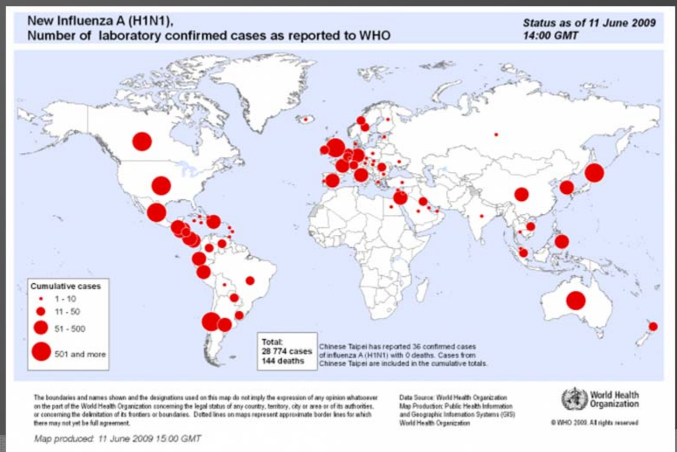
Any model that can be encoded in a matrix over a time sequence !

Signals Processing - Speech Recognition

Demographic Change



Predicting Spread of Infectious disease

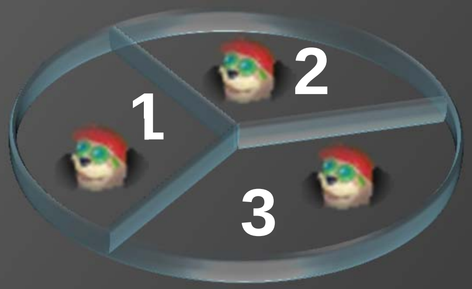


Transition Matrix

```
      [  0  2/3 1/3 ]
  M = [ 1/3  0  2/3 ]
      [ 2/3 1/3  0  ]
```

Measurements @ 75% accuracy

```
  at t1, m2 = NO
   => [ 3/4 1/4 3/4 ]
  at t2, m3 = YES
   => [ 1/4 1/4 3/4 ]
  at t3, m3 = NO
   => [ 3/4 3/4 1/4 ]
```



No Measurement Guess

```
s0 = [ 1 0 0 ]
p1(s1) = s0 x M => [  0  2/3 1/3 ]
p2(s2) = p1 x M => [ .44 .11 .44 ]
p3(s3) = p2 x M => [ 1/3 4/9 2/9 ]
```

Filtering to State Three (normalized)

```
s0 = [ 1 0 0 ]
p1(s1|m1) = s0 x M * m1
 => [ 0 2/5 3/5 ]
p2(s2|m2) = p1 x M * m2
 => [ .3478 .1304 .5217 ]
p3(s3|m3) = p2 x M * m3
 => [ 81/179 84/179 14/179]
```

Smoothing to State Two (normalized)

```
p4 = [ 1 1 1 ]
p3 = p4 * m3
 => [ 3/4 3/4 1/4 ]
p2 = p3 x MT * Filtering p2
 => [ .2381 .1905 .5714 ]
```