Applications of Linear Algebra in Aircraft Stability and Control

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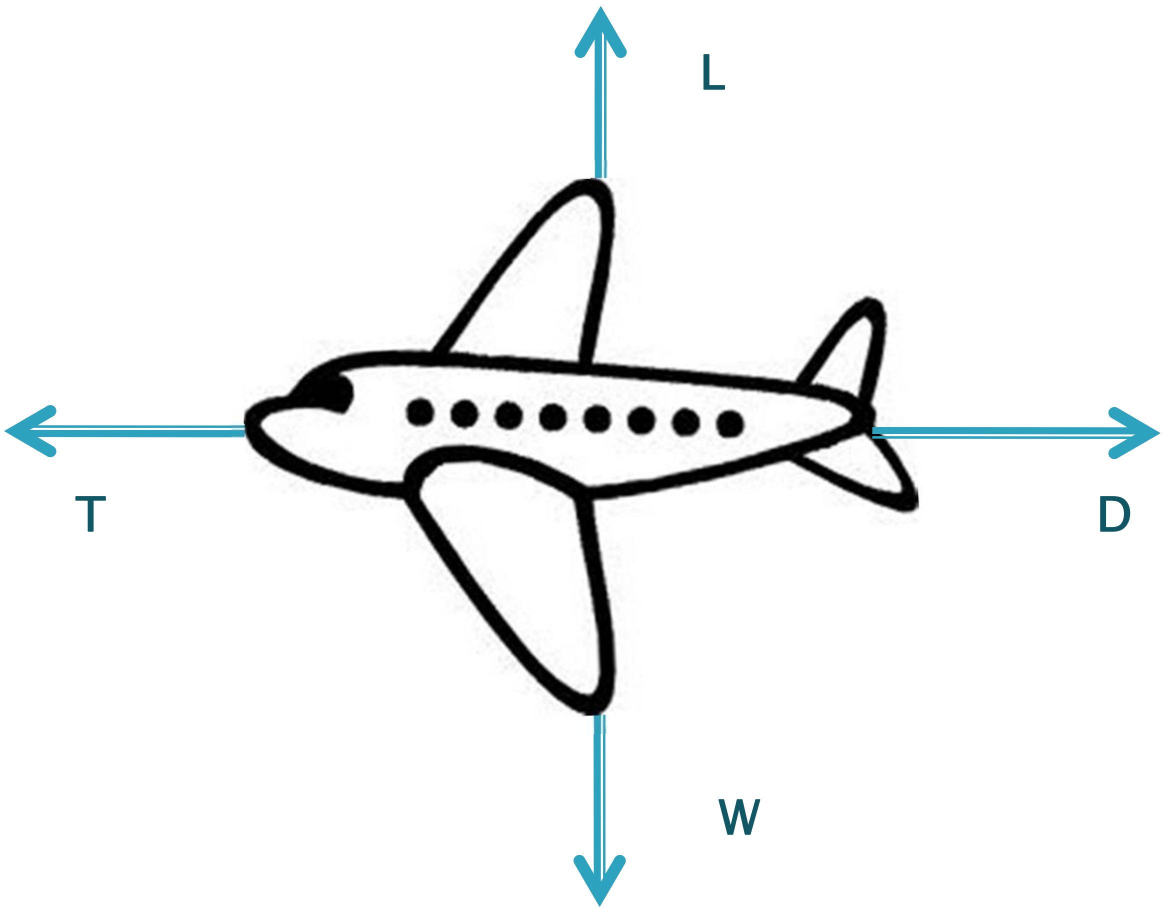
Longitudinal Stability

- Trim

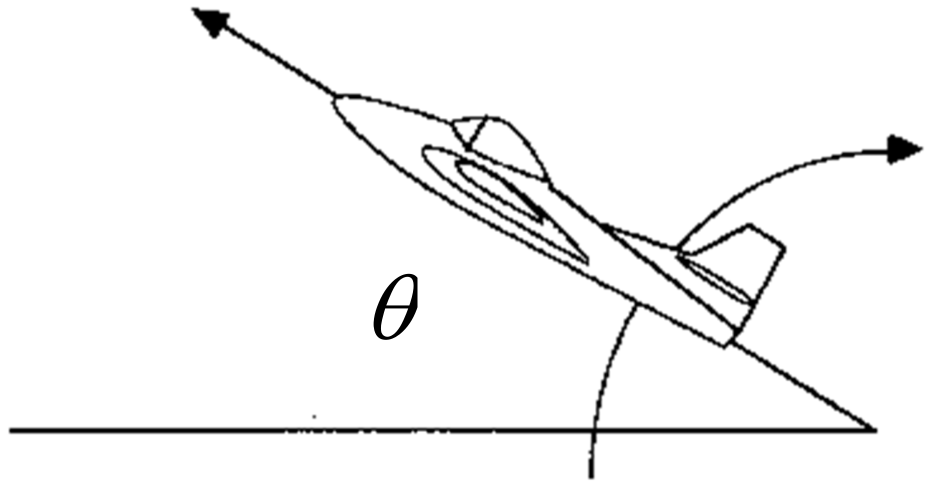
L = W
T = D

Is the ideal
Trim Condition



Longitudinal Stability (cont.)

- Pitch



- Pitch is the only parameter of longitudinal motion in an aircraft, therefore it is the only thing considered when analyzing longitudinal dynamic stability.

Dynamic Stability

$$\dot{x} = Ax + B\eta$$

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}}Z_u & M_w + M_{\dot{w}}Z_w & M_q + M_{\dot{w}}u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_\delta & X_{\delta_T} \\ Z_\delta & Z_{\delta_T} \\ M_\delta + M_{\dot{w}}Z_\delta & M_{\delta_T} + M_{\dot{w}}Z_{\delta_T} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\delta_T \end{bmatrix}$$

Characteristic Equation of A

$$\det(A - \lambda I) = 0$$

- This equation will determine the Eigen-values of A.
- These values will then be analyzed to evaluate the longitudinal dynamic stability of the aircraft.
- For the majority of situations the Eigen-values will be comprised of an imaginary and real component (i.e. $\lambda = -12 \pm 3i$).
- For an aircraft there are three degrees of freedom which corresponds to 6 Eigen-values.

Analysis

$$\lambda = c_1 \pm c_2 i$$

Stable	Not Stable
$c_1 < 0$	$c_1 > 0$

$$c_1 = 0$$

Disturbance

- WARNING: AIRCRAFT APPROACHING STALL!

Controllability

- Controllability is defined as the ability to use control surfaces on a aircraft to bring it back to a stable state.
- For example, if our aircraft has pitched to the point of stall it would become necessary to use the elevator to lift the tail and lower the nose. This will oscillate the system until it converges back to a steady state.

Controllability (cont.)

Determined by controllability matrix:

$$V = [B \quad AB \quad \ldots \quad A^{N-1}B]$$

If the rank of V is in the same order (n-1) as V, then the system is controllable.

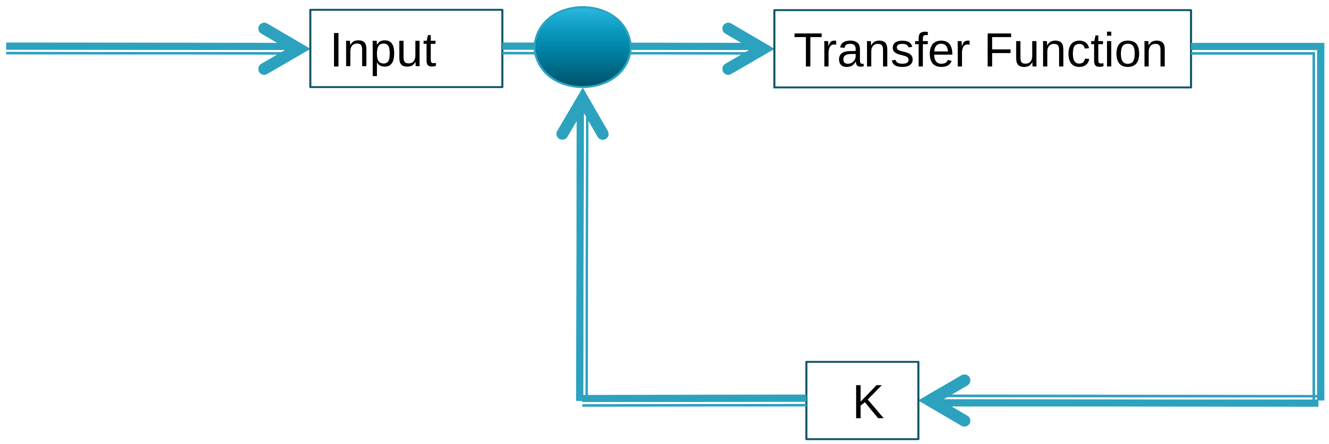
This ensures that the states that are inputted into the system are returned (this will be described more in depth in the augmentation portion)

Augmentation

- Once it is determined that the aircraft has the ability to be controlled, an augmentation system can be applied (i.e. auto-pilot or flight computer)
- These systems operate by analyzing the current situation of the states and adjusting them until the desired state is achieved.

Augmentation (cont.)

- Closed loop system:



- Example: Elevator deflection angle

Augmentation (cont.)

- Re-calculation of dynamic stability:

$$A^{*} = A + Bk^{T}$$

$$\det(A^{*} - \lambda I) = 0$$

Questions?