- derivative ⟷ slope of tangent line ⟷ instantaneous ROC

$m = f'(x)\Big|_{x=a}$

$y - y_0 = m(x - x_0)$

definition difference quotient

formulas
- power rule $\longrightarrow \frac{d}{dx}\Box^n = n\Box^{n-1} \cdot \frac{d}{dx}\Box$
- product rule $\longrightarrow \frac{d}{dx}(1st)(2nd) = (1st)'(2nd) + (2nd)' \cdot (1st)$
- quotient rule $\longrightarrow \frac{d}{dx}\left[\frac{Hi}{Lo}\right] = \frac{Lo\,DHi - Hi\,DLo}{Lo^2}$
- exponential rule $\longrightarrow \frac{d}{dx}e^{\Box} = e^{\Box} \cdot \frac{d}{dx}\Box$
- *Chain rule*

$f - x - t$: $\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt}$

f — x, y — t: $\frac{df}{dt} = f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt}$

Fncs of 1 variable

Give yourself some examples to do:

e.g. differentiate the following:

(a) $g(x) = e^{4-x}$ (b) $f(x) = \frac{2}{x} + \sqrt[3]{x} + \sqrt{5}$

(c) $h(x) = x^2 e^{3x+5}$ (d) $f(x) = \frac{\sqrt{x}}{x^2+1}$

Fncs of 2 variables

e.g. Find the 1st order partial derivatives

(a) $f(x,y) = e^{4y+5x} + x^6 + 7y$

(b) $f(x,y) = \frac{xy}{x^6+7}$

Check your answers with your study buddies!

Business Applications using Chain rule

$\boxed{\text{1 variable}}$

(8 points) When L worker-hours of labor are employed, a factory will produce $Q(L)$ kilograms of a particular commodity, where $Q(L) = 230L^{1/2}$. Suppose that the labor level varies with time in such a way that t months from now, $L(t)$ worker-hours will be employed, where $L(t) = 6t - 0.5t^2 + 9$. At what rate is the production level changing as a function of time 4 months from now? **Round to two decimal places if necessary and include units with your answer.**

Demand is (circle one) increasing/decreasing

Find yourself a 2 variable version problem to do !!

e.g. #3 on Exam 3

$$\begin{array}{c} Q \\ | \\ L \\ | \\ t \end{array}$$

Goal: find $\left.\frac{dQ}{dt}\right|_{t=6}$

Demand is $\uparrow$ if $\left.\frac{dQ}{dt}\right|_{t=6} > 0$, otherwise $\downarrow$

Ans: 115 Kg/month

e.g. #7 on Exam 2

Marginal Analysis

For fnxs of 1 variable

Idea: Δ output $\approx f'(x_0) \cdot \Delta$ input

But in marginal analysis, we always assume Δ input $= 1$

So, Δ output $\approx f'(x_0)$, where x_0 = current standing point

e.g. #5 on Exam 2

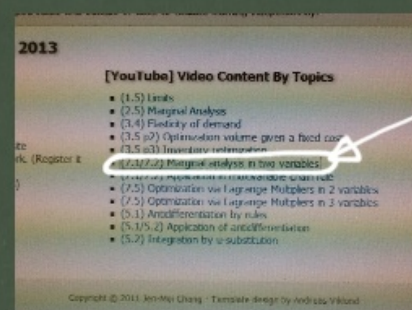
For fnxs of 2 variables

Δ output $\approx f_x \cdot \Delta x + f_y \cdot \Delta y$

In most cases, we fix one variable the same

For example, let $\Delta y = 0$ and allow one unit change in x: $\Delta x = 1$, then

Δ output $\approx f_x(x_0, y_0)$.



here is an example problem

Elasticity of Demand

Remember to use the formula

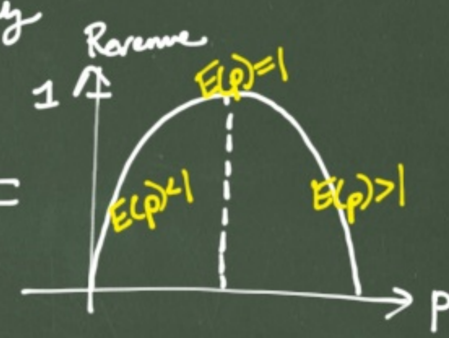
$$E(p) = -\frac{p}{q}\cdot\frac{dq}{dp}$$

this negative sign
means that a 1% ↑ in price
will necessarily cause a 1% ↓ in demand.

Remember to **JUSTIFY** your answer!

After we evaluate $E(p)$ at a given P value, we say

- the demand is <u>elastic</u> if $E(p) > 1$
- " <u>inelastic</u> " < 1
- " <u>unit elastic</u> " $= 1$



Optimization : **1 variable**

1st derivative test

2nd derivative test

only work for non-constrainted problems

e.g. #2 + #6 on Exam 2

Constrainted problems using the concept of absolute extreme on a restricted interval.

e.g. #4 on Exam 2

2 variables

D-test for non-constrainted problems

e.g. #2 on exam 4

Lagrange Multiplies for constrainted problems

e.g. #3 on exam 4

Optimization Word Problems

(Sections 3.5 part 1, 2, 3)

- maximizing yield/profit
  - e.g. #4 on exam 3
    - slide 6, Problem 3 in §3.5 p1 notes
- minimizing cost, maximizing volume/area given some constraint
  - e.g. Problems 1 ~ 5 on §3.5 p2 notes
    - #5 on exam 3
- minimizing cost in an inventory problem
  - e.g. #1 on exam 4

Always remember to **verify** that a min/max is obtained!

Integration / Antidifferentiation

e.g. Integrate the following:

(a) $\int 5t e^{t^2+4}\,dt$

(b) $\int \frac{3x^2+11}{x^3+11x}\,dx$

(c) $\int (x^5+4x)(x^6+12x^2)^3\,dx$

• Remember the basic formulas:

$$k\int x^n\,dx = k\frac{x^{n+1}}{n+1} + C$$

$$k\int x^{-1}\,dx = k\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$$

recipe to Success

• u-substitution

- let u = inner most of the most complicated expression
- then find $\frac{du}{dx}$, solve for dx
- Substitute the original integral with the new expressions so that the new integral is free of x's.
- integrate with respect to u using the basic formulas above
- Change the result back to x's with the definition of u. and $+C$ at the end.

Business Applications requiring antidifferentiation:

e.g #6 on exam 4. It's more likely that you will need to use u-sub. to integrate the function.

Remember to find the "C" using the initial condition !!