

1. The graph of f(x) is shown here.

- (a) Find the domain of f(x) in interval notation.
- (b) Find the zeros and their corresponding multiplicities.
- (c) What is/are the equation(s) of the vertical asymptote(s) of f(x)?
- (d) What is the behavior near the vertical asymptotes? Fill in the blanks.

$$f(x) \to \_$$
 as  $x \to \_$  and  $f(x) \to \_$  as  $x \to \_$   
 $f(x) \to$  as  $x \to$  and  $f(x) \to$  as  $x \to$ 

- (e) What is/are the equation(s) of the horizontal asymptote(s) of f(x)?
- (f) What is the end behavior? Fill in the blanks.

 $f(x) \rightarrow \_\_\_$  as  $x \rightarrow \_\_\_$  and  $f(x) \rightarrow \_\_$  as  $x \rightarrow \_\_\_$ 

- (g) On what interval(s) is  $f(x) \ge 0$ ? Answer in interval notation.
- (h) On what interval(s) is f(x) < 0? Answer in interval notation.

- 2. Find a degree 4 polynomial, g(x) whose graph is shown here. Use correct function notation.

- 3. Consider the polynomial  $P(x) = x^3 4x^2 9x + 36$ .
  - (a) Factor the polynomial. What are the x-intercepts (zeros) of P(x)? What are their corresponding multiplicities?
  - (b) To answer this question, complete the table:
    - Column 1: Enter the *x*-values of all zeros and test points, in order from least to greatest.
    - Column 2: Enter the value of P(x) at the value in Column 1.
    - Column 3: Enter "above," "below" or "on" to indicate if the graph of P(x) is above, below or on the x-axis.

x	P(x)	Is the graph of $P(x)$ above, below, or on the x-axis?
		below, of on the x-axis!

- (c) Use your table to solve the inequality P(x) < 0. Write your answer in interval notation, and graph your answer on the number line.
- (d) Sketch a graph of P(x). Make sure the x- and y-intercepts and end behavior are correct.

- 4. Consider the rational function  $r(x) = \frac{3x+6}{x^2+2x-8}$ .
  - (a) Find the equation(s) of the horizontal asymptote(s), if any. Explain how you know.
  - (b) Determine the end behavior. Fill in the blanks.

 $r(x) \rightarrow \_\_\_$  as  $x \rightarrow \_\_\_$  and  $r(x) \rightarrow \_\_$  as  $x \rightarrow \_\_\_$ 

- (c) Find the coordinates of the *x*-intercepts, if any.
- (d) Find the equation(s) of the vertical asymptote(s), if any.
- (e) Fill out the table below.
  - i. Column 1: Enter the *x*-values of all zeros, vertical asymptotes and test points, in order from least to greatest.
  - ii. Column 2: Enter "+", "-", "0" or "undefined," to indicate the sign of r(x) at the value in Column 1.

iii. Column 3: Enter "zero," "asymptote" or "test point."

Explain below how you obtain the entry in Column 2 for each of your test points.

<i>x</i> -value	+, -, 0, or undefined?	zero, asymptote, or test point?

(f) Determine the behavior near the vertical asymptotes.

 $r(x) \rightarrow \_$  as  $x \rightarrow \_$  and  $r(x) \rightarrow \_$  as  $x \rightarrow \_$  $r(x) \rightarrow \_$  as  $x \rightarrow \_$  and  $r(x) \rightarrow \_$  as  $x \rightarrow \_$ 

(g) Sketch a graph of r(x). Make sure the x- and y-intercepts and end behavior are correct.

- 5. Consider the function  $f(x) = -2x^7 4$ .
  - (a) f(x) is a (circle one) polynomial / rational function.
  - (b) Does f(x) have any asymptotes?
  - (c) What is the domain of f(x)?
  - (d) Fill in the blanks to describe the end behavior of f(x).

 $f(x) \rightarrow \_\_\_$  as  $x \rightarrow \_\_\_$  and  $f(x) \rightarrow \_\_\_$  as  $x \rightarrow \_\_\_$ 

(e) Sketch a graph of f(x). Make sure the end behavior is correct.

6. (a) A function f(x) satisfies all the following.

 $\begin{aligned} f(x) &\to \infty \text{ as } x \to 3^- \\ f(x) &\to -\infty \text{ as } x \to 3^+ \\ f(x) &\to 4 \text{ as } x \to \infty \\ f(x) &\to 4 \text{ as } x \to -\infty \end{aligned}$ 

What information do the first two conditions provide?

What information do the last two conditions provide?

- (b) On a separate set of coordinates, sketch the graph of a function g(x) that satisfies all the following. f(x) → ∞ as x → ∞
  - $f(x) \to \infty$  as  $x \to \infty$   $f(x) \to -\infty$  as  $x \to -\infty$ and f(x) has the following zeros (x-intercepts) with corresponding multiplicities:  $(-1,0) \to m = 2$ ,  $(1,0) \to m = 1$ ,  $(4,0) \to m = 3$ ,  $(9,0) \to m = 1$
- 7. (a) There are infinitely many polynomials of degree 4 that have a zero of multiplicity 2 at x = 3, and zeros of multiplicity 1 at x = 0 and x = 10. Name three of them. Leave your functions in factored form.
  - (b) Only one polynomial fitting the description in part (a) has a graph that passes through the point (2, 120). Find that polynomial.
- 8. (a) For what values of x is  $x^2 \ge 49$ ? Use correct notations.
  - (b) For what values of x is  $x^4 < 16$ ? Use correct notations.