1. The graph of $f(x)$ is shown here.

(a) Find the domain of $f(x)$ in interval notation.
(b) Find the zeros and their corresponding multiplicities.
(c) What is/are the equation(s) of the vertical asymptote(s) of $f(x)$ ?
(d) What is the behavior near the vertical asymptotes? Fill in the blanks.

$$
\begin{aligned}
& f(x) \rightarrow \quad \text { as } x \rightarrow \ldots \quad \text { and } f(x) \rightarrow \quad \text { as } x \rightarrow \\
& f(x) \rightarrow \quad \text { as } x \rightarrow \ldots \quad \text { and } f(x) \rightarrow \quad \text { as } x \rightarrow
\end{aligned}
$$

(e) What is/are the equation(s) of the horizontal asymptote(s) of $f(x)$ ?
(f) What is the end behavior? Fill in the blanks.

$$
f(x) \rightarrow{ }_{\square} \quad \text { as } x \rightarrow \ldots \quad \text { and } f(x) \rightarrow \ldots \quad \text { as } x \rightarrow
$$

(g) On what interval(s) is $f(x) \geq 0$ ? Answer in interval notation.
(h) On what interval(s) is $f(x)<0$ ? Answer in interval notation.
2. Find a degree 4 polynomial, $g(x)$ whose graph is shown here. Use correct function notation.

3. Consider the polynomial $P(x)=x^{3}-4 x^{2}-9 x+36$.
(a) Factor the polynomial. What are the $x$-intercepts (zeros) of $P(x)$ ? What are their corresponding multiplicities?
(b) To answer this question, complete the table:

- Column 1: Enter the $x$-values of all zeros and test points, in order from least to greatest.
- Column 2: Enter the value of $P(x)$ at the value in Column 1.
- Column 3: Enter "above," "below" or "on" to indicate if the graph of $P(x)$ is above, below or on the $x$-axis.

| $x$ | $P(x)$ | Is the graph of $P(x)$ above, <br> below, or on the $x$-axis? |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

(c) Use your table to solve the inequality $P(x)<0$. Write your answer in interval notation, and graph your answer on the number line.
(d) Sketch a graph of $P(x)$. Make sure the $x$ - and $y$-intercepts and end behavior are correct.
4. Consider the rational function $r(x)=\frac{3 x+6}{x^{2}+2 x-8}$.
(a) Find the equation(s) of the horizontal asymptote(s), if any. Explain how you know.
(b) Determine the end behavior. Fill in the blanks.

$$
r(x) \rightarrow{ }_{\sim} \quad \text { as } x \rightarrow \ldots \quad \text { and } r(x) \rightarrow \ldots \quad \text { as } x \rightarrow
$$

(c) Find the coordinates of the $x$-intercepts, if any.
(d) Find the equation(s) of the vertical asymptote(s), if any.
(e) Fill out the table below.
i. Column 1: Enter the $x$-values of all zeros, vertical asymptotes and test points, in order from least to greatest.
ii. Column 2: Enter " + ", "-", " 0 " or "undefined," to indicate the sign of $r(x)$ at the value in Column 1.
iii. Column 3: Enter "zero," "asymptote" or "test point."

Explain below how you obtain the entry in Column 2 for each of your test points.

| $x$-value | ,,+- 0, or undefined? | zero, asymptote, or test point? |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

(f) Determine the behavior near the vertical asymptotes.

$$
\begin{aligned}
& r(x) \rightarrow \quad \text { as } x \rightarrow \ldots \quad \text { and } r(x) \rightarrow \quad \text { as } x \rightarrow Z_{Z} \quad \text { _____ } \\
& r(x) \rightarrow \quad \text { as } x \rightarrow \ldots \quad \text { and } r(x) \rightarrow \quad \text { as } x \rightarrow
\end{aligned}
$$

(g) Sketch a graph of $r(x)$. Make sure the $x$ - and $y$-intercepts and end behavior are correct.
5. Consider the function $f(x)=-2 x^{7}-4$.
(a) $f(x)$ is a (circle one) polynomial / rational function.
(b) Does $f(x)$ have any asymptotes?
(c) What is the domain of $f(x)$ ?
(d) Fill in the blanks to describe the end behavior of $f(x)$.

$$
f(x) \rightarrow \quad \text { as } x \rightarrow \ldots \quad \text { and } f(x) \rightarrow \ldots \quad \text { as } x \rightarrow
$$

(e) Sketch a graph of $f(x)$. Make sure the end behavior is correct.
6. (a) A function $f(x)$ satisfies all the following.
$f(x) \rightarrow \infty$ as $x \rightarrow 3^{-}$
$f(x) \rightarrow-\infty$ as $x \rightarrow 3^{+}$
$f(x) \rightarrow 4$ as $x \rightarrow \infty$
$f(x) \rightarrow 4$ as $x \rightarrow-\infty$

What information do the first two conditions provide?
What information do the last two conditions provide?
(b) On a separate set of coordinates, sketch the graph of a function $g(x)$ that satisfies all the following.
$f(x) \rightarrow \infty$ as $x \rightarrow \infty$
$f(x) \rightarrow-\infty$ as $x \rightarrow-\infty$
and $f(x)$ has the following zeros ( $x$-intercepts) with corresponding multiplicities:
$(-1,0) \rightarrow m=2$,
$(1,0) \rightarrow m=1$,
$(4,0) \rightarrow m=3$,
$(9,0) \rightarrow m=1$
7. (a) There are infinitely many polynomials of degree 4 that have a zero of multiplicity 2 at $x=3$, and zeros of multiplicity 1 at $x=0$ and $x=10$. Name three of them. Leave your functions in factored form.
(b) Only one polynomial fitting the description in part (a) has a graph that passes through the point $(2,120)$. Find that polynomial.
8. (a) For what values of $x$ is $x^{2} \geq 49$ ? Use correct notations.
(b) For what values of $x$ is $x^{4}<16$ ? Use correct notations.

