1. The graph of \( f(x) \) is shown here.

(a) Find the domain of \( f(x) \) in interval notation.

(b) Find the zeros and their corresponding multiplicities.

(c) What is/are the equation(s) of the vertical asymptote(s) of \( f(x) \)?

(d) What is the behavior near the vertical asymptotes? Fill in the blanks.

\[
\begin{align*}
  f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}} & \text{and } f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}} \\
  f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}} & \text{and } f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}}
\end{align*}
\]

(e) What is/are the equation(s) of the horizontal asymptote(s) of \( f(x) \)?

(f) What is the end behavior? Fill in the blanks.

\[
\begin{align*}
  f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}} & \text{and } f(x) &\to \underline{\phantom{0}} & \text{as } x &\to \underline{\phantom{0}} \\
\end{align*}
\]

(g) On what interval(s) is \( f(x) \geq 0 \)? Answer in interval notation.

(h) On what interval(s) is \( f(x) < 0 \)? Answer in interval notation.
2. Find a degree 4 polynomial, \( g(x) \) whose graph is shown here. Use correct function notation.

3. Consider the polynomial \( P(x) = x^3 - 4x^2 - 9x + 36 \).

   (a) Factor the polynomial. What are the \( x \)-intercepts (zeros) of \( P(x) \)? What are their corresponding multiplicities?

   (b) To answer this question, complete the table:
   - Column 1: Enter the \( x \)-values of all zeros and test points, in order from least to greatest.
   - Column 2: Enter the value of \( P(x) \) at the value in Column 1.
   - Column 3: Enter “above,” “below” or “on” to indicate if the graph of \( P(x) \) is above, below or on the \( x \)-axis.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
<th>Is the graph of ( P(x) ) above, below, or on the ( x )-axis?</th>
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   (c) Use your table to solve the inequality \( P(x) < 0 \). Write your answer in interval notation, and graph your answer on the number line.

   (d) Sketch a graph of \( P(x) \). Make sure the \( x \)- and \( y \)-intercepts and end behavior are correct.
4. Consider the rational function \( r(x) = \frac{3x + 6}{x^2 + 2x - 8} \).

(a) Find the equation(s) of the horizontal asymptote(s), if any. Explain how you know.

(b) Determine the end behavior. Fill in the blanks.

\[
r(x) \to \text{ } \text{ as } x \to \text{ } \text{ and } r(x) \to \text{ } \text{ as } x \to \text{ }
\]

(c) Find the coordinates of the \( x \)-intercepts, if any.

(d) Find the equation(s) of the vertical asymptote(s), if any.

(e) Fill out the table below.

i. Column 1: Enter the \( x \)-values of all zeros, vertical asymptotes and test points, in order from least to greatest.

ii. Column 2: Enter “+”, “−”, “0” or “undefined,” to indicate the sign of \( r(x) \) at the value in Column 1.

iii. Column 3: Enter “zero,” “asymptote” or “test point.”

Explain below how you obtain the entry in Column 2 for each of your test points.

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>+, −, 0, or undefined?</th>
<th>zero, asymptote, or test point?</th>
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(f) Determine the behavior near the vertical asymptotes.

\[
r(x) \to \text{ } \text{ as } x \to \text{ } \text{ and } r(x) \to \text{ } \text{ as } x \to \text{ }
\]

\[
r(x) \to \text{ } \text{ as } x \to \text{ } \text{ and } r(x) \to \text{ } \text{ as } x \to \text{ }
\]

(g) Sketch a graph of \( r(x) \). Make sure the \( x \)- and \( y \)-intercepts and end behavior are correct.

5. Consider the function \( f(x) = -2x^7 - 4 \).

(a) \( f(x) \) is a (circle one) polynomial / rational function.

(b) Does \( f(x) \) have any asymptotes?

(c) What is the domain of \( f(x) \)?

(d) Fill in the blanks to describe the end behavior of \( f(x) \).

\[
f(x) \to \text{ } \text{ as } x \to \text{ } \text{ and } f(x) \to \text{ } \text{ as } x \to \text{ }
\]

(e) Sketch a graph of \( f(x) \). Make sure the end behavior is correct.
6. (a) A function \( f(x) \) satisfies all the following.
\[
\begin{align*}
f(x) &\to \infty \text{ as } x \to 3^- \\
f(x) &\to -\infty \text{ as } x \to 3^+ \\
f(x) &\to 4 \text{ as } x \to \infty \\
f(x) &\to 4 \text{ as } x \to -\infty
\end{align*}
\]
What information do the first two conditions provide?

What information do the last two conditions provide?

(b) On a separate set of coordinates, sketch the graph of a function \( g(x) \) that satisfies all the following.
\[
\begin{align*}
f(x) &\to \infty \text{ as } x \to \infty \\
f(x) &\to -\infty \text{ as } x \to -\infty
\end{align*}
\]
and \( f(x) \) has the following zeros (x-intercepts) with corresponding multiplicities:
\[
\begin{align*}
(-1,0) &\to m = 2, \\
(1,0) &\to m = 1, \\
(4,0) &\to m = 3, \\
(9,0) &\to m = 1
\end{align*}
\]

7. (a) There are infinitely many polynomials of degree 4 that have a zero of multiplicity 2 at \( x = 3 \), and zeros of multiplicity 1 at \( x = 0 \) and \( x = 10 \). Name three of them. Leave your functions in factored form.

(b) Only one polynomial fitting the description in part (a) has a graph that passes through the point \((2,120)\). Find that polynomial.

8. (a) For what values of \( x \) is \( x^2 \geq 49 \)? Use correct notations.

(b) For what values of \( x \) is \( x^4 < 16 \)? Use correct notations.