Practice Test 2

1. Let \( f(x) = 4x^2 + 40x + 91 \).
   
   (a) Without changing into Standard Form, what is the vertex of the parabola \( f(x) \)?
   
   (b) Complete the square to rewrite the function \( f(x) \) in Standard Form.
   
   (c) Using the Standard Form, find the vertex of \( f \) and confirm that it agrees with your answer from part (a).
   
   (d) The parabola \( f(x) \) opens upward / downward (circle one).
   
   (e) Starting from the Polynomial Form, \( f(x) = 4x^2 + 40x + 91 \), find the discriminant \( D = b^2 - 4ac \). The value of the discriminant tells us that there are
   
   no \( x \)-intercepts / one \( x \)-intercept / two \( x \)-intercepts (circle one).
   
   (f) The location of the vertex found in parts (a) and (c) in combination with the answer from part (d) should confirm the number of \( x \)-intercepts. Does it?
   
   (g) Use the Standard Form to solve for the \( x \)-intercepts. Write your answer(s) as an (ordered, pair).
   
   (h) Use the Polynomial Form to solve for the \( y \)-intercept. Write your answer as an (ordered, pair).
   
   (i) Graph the parabola \( f(x) \).

   (For kicks) Use the graph (and specifically, the \( x \)-intercepts) to solve the equation
   
   \[ 4 \left( \frac{1 + t}{4} \right)^2 + 40 \left( \frac{1 + t}{4} \right) + 91 = 0 \]

2. The graph of the quadratic function \( g(x) \) obtains its minimum value of \(-1\) at \( x = -3 \). Additionally, its \( y \)-intercept is the point \((0, 17)\). Find a formula for \( g(x) \). For full credit, show your work and use correct notation.

3. The graph of the quadratic function \( g(x) \) is shown here. Find a formula for \( g(x) \).
Word Problems:

(Just find the formula)

4. A rectangular container has a square base with side length \( x \) and a volume of \( 80 \text{ft}^3 \). Find a formula for the container’s surface area in terms \( x \). (For additional practice, review all problems in WebAssign Sets 7 and 8.)

5. Imagine a cylinder with height \( h \) four times the radius of its base \( r \), as shown in Figure A (not to scale).

(a) Find the formula for the volume of the cylinder in terms of its radius, \( r \). (Use the correct function notation)

(b) Find the formula for the volume of the cylinder in terms of its height, \( h \). (Use the correct function notation)

(Use the given formula to solve a problem)

6. Sarah shoots a pellet gun straight upward so that the height of the pellet (in feet) after \( t \) seconds is given by the formula, \( h(t) = 160t - 16t^2 \).

(a) How long does it take the pellet to reach its maximum height? Show your work and include units with your answer.

(b) What is the pellet’s maximum height? Show your work and include units with your answer.

(Find the formula and use it to solve a problem)

7. A farmer has 2800 ft of fencing material and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river. (See Figure B)

(a) What are the dimensions of the field of largest area that he can fence?

(b) What is the largest area that he can fence?
8. Solve the following equations. Give exact answers (do not round). If there are no real solutions, say so.

(a) \(x^{\frac{1}{5}} = -6\)  \hspace{1cm} (b) \(x^{\frac{1}{5}} = 6\)  \hspace{1cm} (c) \(x^{\frac{1}{2}} = -3\)  \hspace{1cm} (d) \(x^{\frac{1}{2}} = 3\)

(e) \(x^4 = -2\)  \hspace{1cm} (f) \(x^4 = 5\)  \hspace{1cm} (g) \(x^5 = -3\)  \hspace{1cm} (h) \(x^5 = 7\)

9. Solve the equation \(x^{12} + x^6 - 30 = 0\). Give exact answers (do not round). If there are no real solutions, say so.

10. Consider the polynomial \(p(x) = -5x^7 + 3x^4 + 7x^3 - 9x - 11\).

(a) Evaluate and simplify \(p(2x)\).

(b) Evaluate and simplify \(p(-x)\).

11. Write the formula for the function \(h(x)\) whose graph is the same as \(g(x) = -4x + 7\), except shifted up by 3 and to the left by 5. Simplify your answer.

12. The graph of the functions \(A(x)\) and \(g(x)\) are shown below.

(a) The function \(A(x)\) can be obtained from the function \(g(x)\) by applying two transformations. What are they (in words)?

(b) Sketch a graph of \(B(x) = -g(x - 10)\). Label the graph \(B(x)\).

(c) Let \(C(x)\) be the function whose graph is the same as the graph of \(g(x)\), except stretched vertically by a factor of 2, and shifted 5 units to the left. Sketch a graph of \(C(x)\). Label it \(C(x)\).

(d) Write a formula for the function \(C(x)\) in terms of \(g(x)\).