Student learning objectives
- Relate changes in the formula for a function to changes in the function’s graph.
- Use function notation correctly.

Example
Find a formula for each parabola.

Vertex of \( g(x) \):

Another point on the graph of \( g(x) \):

Vertex of \( f(x) \):

Another point on the graph of \( f(x) \):

What can we do to the graph of \( g \) to make it into the graph of \( f \) ?

Fill in blank:
The graph of \( f \) is the same as the graph of \( g \), except shifted \( \square \) by \( \square \) units, and \( \square \) by \( \square \) units.

Write a formula for \( f(x) \) in terms of \( g(x) \): ____________________________

Example
Write a formula for the function \( f(x) \) whose graph is the same as the graph of \( g(x) \), except shifted down by 6 units and to the left by 5 units.

Should the letter \( g \) appear in your formula? Yes / No

Write a formula for the function \( f(x) \) whose graph is the same as the graph of \( g(x) = \frac{4}{x^2} \), except shifted down by 6 units and to the left by 5 units.

Should the letter \( g \) appear in your formula? Yes / No
Match the following:
The graph of $f$ is the same as the graph of $g$, except shifted…

1. up by 5 units
2. down by 5 units
3. left by 5 units
4. right by 5 units
5. right by 5 units and up by 5
6. left by 5 units and down by 5

A. $f(x) = g(x + 5)$
B. $f(x) = g(x + 5) - 5$
C. $f(x) = g(x - 5) + 5$
D. $f(x) = g(x) + 5$
E. $f(x) = g(x) - 5$
F. $f(x) = g(x - 5)$

Example (Reflection across the y-axis)

What can we do to the graph of $g$ to make it into the graph of $f$?

Fill in blank: The graph of $f$ is the same as the graph of $g$, except

______________________ across the

______________________.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
<th>$x$</th>
<th>$g(x)$</th>
<th>$f(x)$ in terms of $g(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>10</td>
<td>-1</td>
<td>-1</td>
<td>$f(-5) = g(_______)$</td>
</tr>
<tr>
<td>-4</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>$f(-4) = g(_______)$</td>
</tr>
<tr>
<td>-3</td>
<td>4.5</td>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>5.5</td>
<td>2</td>
<td>5.5</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
<td>3</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>5</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

Write a formula for $f(x)$ in terms of $g(x)$:

$$f(100) = g(_______)$$
Example

Write a formula for the function \( f(x) \) whose graph is the same as the graph of \( g(x) = \frac{4}{x^2} \), except reflected across the \( y \)-axis.

Example (Reflection across the \( x \)-axis)

On the graph below, sketch the graph of the function \( f(x) \) whose graph is the reflection of the graph of \( g(x) \) across the \( x \)-axis.

Should the letter \( g \) appear in your formula? **Yes** / **No**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( x )</th>
<th>( f(x) )</th>
<th>( \ldots ) in terms of ( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>0</td>
<td>-5</td>
<td></td>
<td>( f(-5) = -g(-5) )</td>
</tr>
<tr>
<td>-3</td>
<td>3</td>
<td>-3</td>
<td></td>
<td>( f(-4) = g(___) )</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
<td>0</td>
<td></td>
<td>( \phantom{f(___)} ) ( \phantom{g(___)} )</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td></td>
<td>( \phantom{f(___)} ) ( \phantom{g(___)} )</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
<td>( \phantom{f(___)} ) ( \phantom{g(___)} )</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td></td>
<td>( f(100) = g(___) )</td>
</tr>
</tbody>
</table>
Example
Write a formula for the function $f(x)$ whose graph is the same as the graph of $g(x) = \frac{4}{x^2}$, except reflected across the $x$-axis.

Check Your Understanding
Give a verbal description on how you can get the graph of $h$ from the graph of $g$.

1. $h(x) = -g(x+3)$

2. $h(x) = 2+g(x-4)$

3. $h(x) = g(-x-2)+3$

Example (Vertical shrink and stretch)
Complete the missing entries.

| $x$ | $g(x) = |x|$ | $x$ | $f(x) = 2|x|$ | $x$ | $h(x) = \frac{1}{2}|x|$ |
|-----|-------------|-----|---------------|-----|-------------------|
| -4  | -4          | -4  | -8            | -4  | -2               |
| -3  | -3          | -3  | -6            | -3  | -1.5             |
| -2  | 2           | -2  | 4             | -2  | 1                |
| -1  | 1           | -1  | 2             | -1  | 0.5              |
| 0   | 0           | 0   | 0             | 0   | 0                |
| 1   | 1           | 1   | 2             | 1   | 0.5              |
| 2   | 2           | 2   | 4             | 2   | 1                |
| 3   | 3           | 3   | 6             | 3   | 1.5              |
| 4   | 4           | 4   | 8             | 4   | 2                |
Write a formula for $f(x)$ and $h(x)$ in terms of $g(x)$.

Graph all three functions on the coordinate axis.

Note that multiplying $g(x)$ by 2 makes the graph (circle one) stretch / shrink vertically by a factor of ____________.

Example (Horizontal shrink and stretch)

Graph the function $f(x)$ whose graph is the same as the graph of $g(x)$ except shrunk horizontally by a factor of 1/2.

Fill in the blanks using your graph.

$f(-1)$ is the same as $g$ (____).
$f(2)$ is the same as $g$ (____).

In notation:

$f(-2) = g(____)
$f(-1) = g(____)
$f(0) = g(____)
$f(1) = g(____)
$f(2) = g(____)

Conclusion: The function $f(x) = g(_______)$ has the same graph as $g(x)$ except shrunk horizontally by a factor of 1/2.

Example

Write the formula for a function $h(x)$ whose graph you can get from the graph of $g(x)$ by stretching horizontally by a factor of 2.
Example

Write a formula for the function \( f(x) \) whose graph is the same as the graph of \( g(x) = \frac{4}{x^2} \), except shrunk horizontally by a factor of 1/3.  

Write a formula for the function \( f(x) \) whose graph is the same as the graph of \( g(x) = \frac{4}{x^2} \), except shrunk horizontally by a factor of 1/3.  Simplify.

Check it out: For this function, shrinking the graph of \( g(x) \) horizontally by a factor of 1/3 gives the same picture as shrinking the graph vertically by a factor of ________.

Summary

Complete the table.

<table>
<thead>
<tr>
<th>To get the graph of ( f(x) ) from ( g(x) ), you ...</th>
<th>Formula for ( f(x) )</th>
<th>In words, to get a formula for ( f ), you ... (In your answer, refer to the input or output of ( g ).)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift the graph of ( g(x) ) up by 2</td>
<td>( f(x) = g(x) + 2 )</td>
<td>add 2 to the output of ( g )</td>
</tr>
<tr>
<td>Shift the graph of ( g(x) ) down by 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shift the graph of ( g(x) ) left by 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shift the graph of ( g(x) ) right by 5</td>
<td>( f(x) = g(x - 5) )</td>
<td>subtract 5 from the input of ( g )</td>
</tr>
<tr>
<td>Reflect the graph of ( g(x) ) across the x-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflect the graph of ( g(x) ) across the y-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stretch the graph of ( g(x) ) vertically by a factor of 6</td>
<td>( f(x) = (1/7)g(x) ) or ( f(x) = g(x)/7 )</td>
<td>multiply the output of ( g ) by 1/7 or divide the output of ( g ) by 7</td>
</tr>
<tr>
<td>Shrink the graph of ( g(x) ) vertically by a factor of 1/7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stretch the graph of ( g(x) ) horizontally by a factor of 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shrink the graph of ( g(x) ) horizontally by a factor of 1/9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Even Functions**

Definition: A function $f$ is called an **even** function if

**Odd Functions**

Definition: A function $f$ is called an **odd** function if