1. (52 points) Answer the following questions for the quadratic function (a) \( f(x) = 2x^2 - 20x + 57 \) and (b) \( f(x) = -x^2 + x + 2 \).

   i (9 points) Convert \( f(x) \) into its standard form (by completing the square). What is its vertex?

   ii (4 points) Find the \( x \)-intercepts of \( f \), if any.

   iii (4 points) Find the \( y \)-intercepts of \( f \), if any.

   iv (5 points) Sketch the graph of \( f \).

   v (4 points) Complete the sentence: The function \( f \) has a (circle one) maximum/minimum at \( \underline{\text{}} \). This graph opens (circle one) upward/downward.

2. (12 points) The graph of the quadratic function \( g(x) \) is shown here. Find a formula for \( g(x) \).

3. (12 points) Find all real solutions of the equation

   \[
   1 + \frac{2x}{(x + 3)(x + 4)} = \frac{2}{x + 3} + \frac{4}{x + 4}.
   \]

4. (12 points) Find a function whose graph is a parabola with vertex \((1, -2)\) and passes through the point \((4, 16)\).

5. (12 points) A soft-drink vendor at a popular beach analyzes his sales records and finds that if he sells \( x \) cans of soda pop in one day, his profit (in dollars) is given by

   \[ P(x) = -0.001x^2 + 3x - 1800. \]

   What is his maximum profit per day and how many cans must he sell for maximum profit? Write your answer in complete sentences.