Group #: _____ Name: _____

1. (5 points) When does a rational function have a slant asymptote?

2. (5 points) Is it okay for parts of a rational function to cross through a slant asymptote? Why or why not?

3. (25 points each) Find the **slant asymptotes** and the **vertical asymptotes**. Examine the function behavior around the function zeros and the x-intercept of the vertical asymptotes. Then, use the information in the *behavior chart* to **sketch a graph** of the given rational function. If necessary, plot additional points to ensure the accuracy of the graph. If available, verify your graph with a computer or a calculator.

   (a) $r(x) = \dfrac{x^2}{x-2}$

   (b) $r(x) = \dfrac{x^2-2x-8}{x}$

   (c) $r(x) = \dfrac{x^3+x^2}{x^2-4}$

4. (15 points) Graph a function $f(x)$ with the following properties.

   - $f(x)$ has a slant asymptote $y = \frac{1}{2}x - \frac{1}{4}$.
   - The only vertical asymptotes of $f(x)$ are $x = -1$ and $x = \frac{1}{2}$.
   - The only x-intercept of $f(x)$ is $(-\sqrt[3]{4}, 0)$.
   - $f(x)$ has the following behavior near the vertical asymptotes:

$$f(x) \to \infty \quad \text{as} \quad x \to -1^- \qquad \text{and} \qquad f(x) \to -\infty \quad \text{as} \quad x \to -1^+$$
$$f(x) \to -\infty \quad \text{as} \quad x \to \frac{1}{2}^- \qquad \text{and} \qquad f(x) \to \infty \quad \text{as} \quad x \to \frac{1}{2}^+$$