Math 113

Group Quiz 8 Solutions

(a) Known

Material: 2400 ft

\[ x + x + y = 2400 \]

\[ 2x + y = 2400 \quad [5] \]

Solve for \( y \)

\[ y = 2400 - 2x \quad \text{substitute} \]

(b) Want to know

\[ A = x \cdot y \]

\[ = x(2400 - 2x) \quad [5] \]

\[ = 2400x - 2x^2 \]

\[ A = -2x^2 + 2400x \quad [12] \]

part (b)

(c) Want to maximize area; we know \( A = -2x^2 + 2400x \) is a parabola opening downward, thus it attains its maximum at

\[ x = -\frac{b}{2a} = -\frac{2400}{2(-2)} = 600 \quad [10] \]

The dimensions that maximize the area are \( x = 600 \) feet and \( y = 1200 \) feet.

[3] Need units!
\( L = 26 \text{ cm} \)

\[
\frac{L - x}{x} = \frac{26 - x}{x}
\]

a) \ \text{known:}

\( \text{side of 1st square} = \frac{x}{4} \rightarrow A_1 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16} \) \[4\]

\( \text{side of 2nd square} = \frac{26-x}{4} \rightarrow A_2 = \left(\frac{26-x}{4}\right)^2 = \frac{676 - 52x + x^2}{16} \) \[4\]

want \( A = A_1 + A_2 \)

\[
A = \frac{x^2}{16} + \frac{676 - 52x + x^2}{16}
\]

\[
= \frac{1}{16} \left(2x^2 - 52x + 676\right)
\]

\[
= \frac{2}{16} \left(x^2 - 26x + 338\right)
\]

\[
= \frac{1}{8} \left(x^2 - 26x + 338\right)
\]

\[
A = \frac{1}{8} x^2 - \frac{13}{4} x + \frac{169}{4}
\]

or \[5\]

b) \ Find the value of \( x \) that minimizes the total area of the 2 squares.

\( A \) is a parabola opening up, so we know it attains its minimum at the vertex where \( x = -\frac{b}{2a} = -\left(\frac{-13}{4}\right) \) \[8\]

\[
x = \frac{13}{4} \cdot \frac{4}{1} = 13 \text{ cm} \] [2 units]

This means that if you cut the wire in half then make two squares, you will attain the minimum area for the total of 2 squares.
Known
\[ x + x + x + x + y + y = 750 \, \text{ft} \]
\[ 5x + 2y = 750 \, \text{ft}. \quad [3] \]

Solve for \( y \):
\[ 2y = \frac{750 - 5x}{2} \]
\[ y = 375 - \frac{5}{2}x \quad \text{substitute} \]

Want to know
\[ A = x \cdot y \]
\[ A = x \left( 375 - \frac{5}{2}x \right) \quad [2] \]
\[ A = 375x - \frac{5}{2}x^2 \]
\[ A = -\frac{5}{2}x^2 + 375x \quad [3] \]

Goal: maximize \( A \).

We know \( A = -\frac{5}{2}x^2 + 375x \) is a parabola which opens downward.
Thus \( A \) attains a maximum at the vertex where \( x = \frac{-b}{2a} \)
\[ x = \frac{-b}{2a} = \frac{-375}{2 \left( \frac{5}{2} \right)} = \frac{375}{5} = 75 \, \text{ft} \quad [3] \]

The largest possible area, produced when \( x = 75 \), is \[ A = \frac{-5}{2} (75)^2 + 375(75) = -14062.5 + 28125 = 14062.5 \, \text{ft}^2 \quad [3] \]

Note that the question does not ask for the dimensions of the fences, but you can use the extra information to check your answer.
\[ x = 75 \, \text{ft} \rightarrow y = 375 - \frac{5}{2}(75) = 187.5 \, \text{ft} \]
\[ A = xy = (75)(187.5) = 14062.5 \, \text{ft}^2 \]
given: fence by road: $5/ft
fence not by road: $3/ft
Area: 1200 ft²

Known: \( A = x \times y = 1200 \text{ ft}^² \)

solve for \( y \)

\[
y = \frac{1200}{x} \quad [3]
\]

Want to know
Cost in terms of \( x \).

\[
C = 3 \cdot y + 3 \cdot x + 3 \cdot y + 5 \cdot x \quad [6]
\]

\[
C = 6y + 8x
\]

\[
C(x) = \frac{7200}{x} + 8x \quad [6]
\]

Units will be $.

\[
V(x) = (20 - 2x)(12 - 2x)(x)
\]

\[
V(x) = (240 - 40x - 24x + 4x²) \cdot x
\]

\[
V(x) = 4x³ - 64x² + 240x \quad [2]
\]

Units will be in $^³$. 