Math 113

Group Quiz 8 Solutions

1. (a)

known

material: 2400 ft

$x + x + y = 2400$

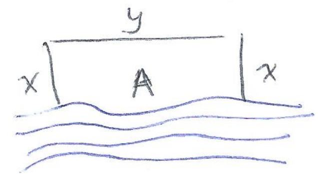
part (a) $\boxed{2x + y = 2400}$ [5]

$\hookrightarrow y = 2400 - 2x \xrightarrow{\text{substitute}}$

solve for y

want to know

(b) A in terms of x



$A = x \cdot y$

$= x(2400 - 2x)$ [5]

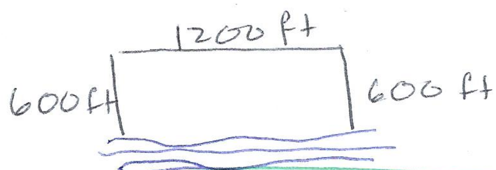
$= 2400x - 2x^2$

$\boxed{A = -2x^2 + 2400x}$ [2]

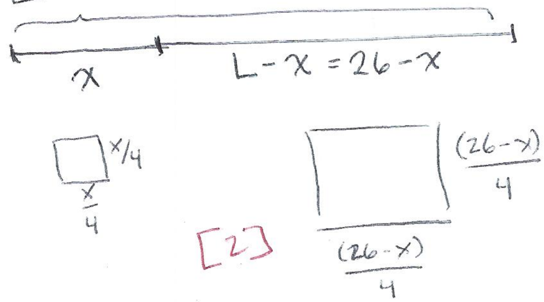
part (b)

c) want to maximize Area; we know $A = -2x^2 + 2400x$ is a parabola opening downward, thus it attains its maximum at $x = -\dfrac{b}{2a} = \dfrac{-2400}{2(-2)} = 600$. [10]

The dimensions that maximize the area are $x = 600$ feet and $y = 1200$ feet [3] need units!



(2) $L = 26$ cm



[2]

a) Known:

side of 1st square $= x/4 \longrightarrow A_1 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$ [4]

Side of 2nd square $= \frac{(26-x)}{4} \longrightarrow A_2 = \left(\frac{26-x}{4}\right)^2 = \frac{676 - 52x + x^2}{16}$ [4]

want $A = A_1 + A_2$

$$A = \frac{x^2}{16} + \frac{676 - 52x + x^2}{16}$$

$$= \frac{1}{16}\left(2x^2 - 52x + 676\right)$$

$$= \frac{2}{16}\left(x^2 - 26x + 338\right)$$

$$= \boxed{\frac{1}{8}\left(x^2 - 26x + 338\right)}$$ or

$$A = \boxed{\frac{1}{8}x^2 - \frac{13}{4}x + \frac{169}{4}}$$ [5]

b) Find the value of x that minimizes the total area of the 2 squares

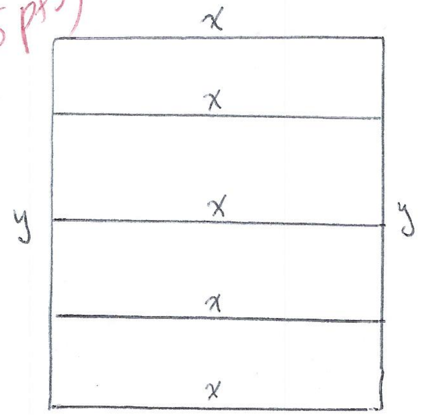
A is a parabola opening up, so we know it attains its minimum at the vertex where $x = \frac{-b}{2a} = \frac{-\left(\frac{-13}{4}\right)}{2\left(\frac{1}{8}\right)}$ [8]

$$\boxed{x = \frac{\frac{13}{4}}{\frac{1}{4}} = \frac{13}{4} \cdot \frac{4}{1} = 13 \text{ cm}}$$ [2 units]

This means that if you cut the wire in half then make two squares, you will attain the minimum area for the total of these two squares together

27

(3) (15 pts) 750 ft of fencing



Known

$x+x+x+x+x+y+y = 750$ ft

$5x+2y = 750$ ft. [3]

Solve for y:

$\frac{2y}{2} = \frac{750}{2} - \frac{5x}{2}$

$y = 375 - \frac{5}{2}x$ substitute

Want to know

A in terms of x

$A = x \cdot y$

$A = x(375 - \frac{5}{2}x)$ [2]

$A = 375x - \frac{5}{2}x^2$

$A = -\frac{5}{2}x^2 + 375x$

[3]

Goal: maximize A.

We know $A = -\frac{5}{2}x^2 + 375x$ is a parabola which opens downward.

Thus A attains a <u>maximum</u> at the vertex where $x = -\frac{b}{2a}$

$x = -\frac{b}{2a} = \frac{-375}{\cancel{2}(\frac{5}{\cancel{2}})} = \frac{375}{5} = 75$ ft [3]

The largest possible area, produced when $x = 75$, is [3]

[1 units]

$A = -\frac{5}{2}(75)^2 + 375(75) = -14062.5 + 28125 = \boxed{14062.5 \text{ ft}^2}$

note that the question does not ask for the dimensions of the fences, but you can use the extra information to check your answer.

$x = 75$ ft $\rightarrow y = 375 - \frac{5}{2}(75) = 187.5$ ft

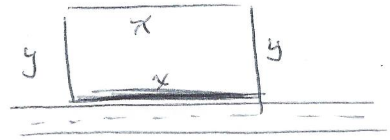
$A = xy = (75)(187.5) = 14062.5$ ft^2

3]

(4) (15 pts) given: fence by road: \$5/ft

fence not by road: \$3/ft

Area: $1200\,ft^2$



Known: $A = x \cdot y = 1200\,ft^2$

solve for y

$y = \frac{1200}{x}$ [3]

substitute

want to know

Cost in terms of x.

$C = 3 \cdot y + 3 \cdot x + 3 \cdot y + 5 \cdot x$ [6]

this comes from $5\frac{\$}{ft} \cdot x\,ft = 5x(\$)$

$C = 6y + 8x$

$= 6\left(\frac{1200}{x}\right) + 8x$

$\boxed{C(x) = \frac{7200}{x} + 8x}$ [6]

units will be \$

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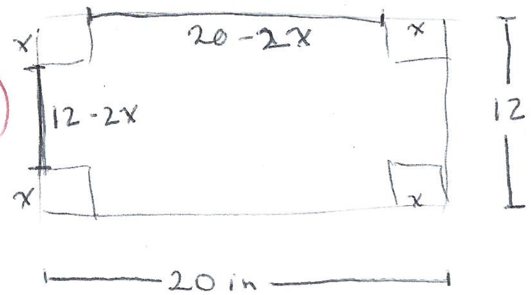
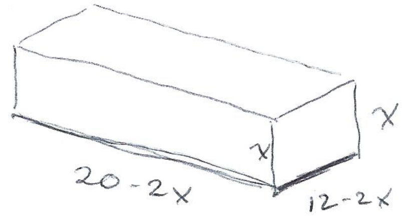
(5) (20 pts)





[10]

$V(x) = (20-2x)(12-2x)(x)$ [8]

$= (240 - 40x - 24x + 4x^2)\,x$

$\boxed{V(x) = 4x^3 - 64x^2 + 240x}$ [2]

units will be in^3