Math 113
Group Quiz 7 Solutions

1) Rectangular building: 3 times as long as it is wide \( l = 3w \)
Find a function that models the area \( A \) in terms of width \( w \).

- Known
  \[ l = 3w \quad [8] \]
- Want to know
  \[ A \text{ in terms of } w \]
  \[ A = l \cdot w \]
  \[ A = (3w) \cdot w \]
  \[ A = 3w^2 \quad [2] \]

2) Rectangle has area of 16 m\(^2\). Find function to model perimeter \( P \) in terms of length \( x \) of one of its sides.

- Known
  \[ A = 16 \text{ m}^2 = x \cdot y \]
  \[ y = \frac{16}{x} \quad [8] \]
- Want to know
  \[ P \text{ in terms of } x \]
  \[ P = 2x + 2y \]
  \[ P = 2x + 2 \left( \frac{16}{x} \right) \]
  \[ P = 2x + \frac{32}{x} \quad [10] \]
  \[ P = 2 \left( \frac{x^2 + 16}{x} \right) \quad [2] \]

Side note:
The units for the perimeter will be meters.
3) height of cylinder is four times the radius \( h = 4r \)

Model Vol of cylinder in terms of \( r \).

\[
\begin{align*}
\text{known} & & \text{Want to know} \\
 h = 4r & & V \text{ in terms of } r. \\
 V = \pi r^2 \cdot h & = \pi r^2 (4r) \quad [10] \\
 V = 4\pi r^3 & \quad [27]
\end{align*}
\]

4) box with volume 60 ft\(^3\) has a square base.

Want: SA in terms of \( x \) (length of one side of base)

\[
\begin{align*}
\text{known} & & \text{Want to know} \\
 V = 60 \text{ft}^3 & = x \cdot x \cdot y \quad \text{solve for } y \\
 y = \frac{60}{x^2} & \quad [8] \\
 \text{or substitute} \\
 \text{SA} = 2 \cdot (x \cdot x + x \cdot y + x \cdot y) & = 2 \cdot (x^2 + 2xy) \quad [10] \\
 & = 2 \left( x^2 + 2x \cdot \left( \frac{60}{x^2} \right) \right) \\
 & = 2 \left( x^2 + \frac{120}{x} \right) \\
 & = 2 \left( \frac{x^3 + 120}{x} \right) \quad [27]
\end{align*}
\]
Two numbers whose sum is 19 → \( x + y = 19 \)

Product is maximized → let \( P = xy \)

(want to maximize \( P \))

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\[ \begin{array}{|c|c|c|} \hline
1 & 18 & 18 \\
2 & 17 & 2 \times 17 = 34 \\
3 & 16 & 3 \times 16 = 48 \\
4 & 15 & 4 \times 15 = 60 \\
5 & 14 & 5 \times 14 = 70 \\
6 & 13 & 6 \times 13 = 78 \\
7 & 12 & 7 \times 12 = 84 \\
8 & 11 & 8 \times 11 = 88 \\
9 & 10 & 9 \times 10 = 90 \\
\hline \end{array} \]

Notice that if I continue the table, the numbers will repeat. \( x \) and \( y \) are chosen arbitrarily, so we can stop here.

Estimated answer: \( x = 9, y = 10 \) will add to 19 and maximize the product \( P = xy \).

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b) Model product in terms of \( x \).

\( x + y = 19 \)  

Want to know \( P \) in terms of \( x \).

\( y = 19 - x \)

\[ P = xy \]

\[ P = x(19 - x) \]

\[ P = 19x - x^2 \]
c) \[ P = 19x - x^2 \quad \text{(from part b)} \]

\[ P = -x^2 + 19x \quad \text{this is a quadratic with a negative} \]

\[ \text{coefficient in front} \]

\[ \rightarrow \text{this means it will graph as} \]

\[ \text{a parabola opening downward} \]

\[ \rightarrow \text{this means it will attain its} \]

\[ \text{maximum at the vertex} \left( \frac{-b}{2a}, \frac{c-b^2}{4a} \right) \]

We only care about the \( x \)-value of the vertex since we want to know what \( x \)-value forces the product to be a maximum.

thus if \[ P = -1 \cdot x^2 + 19x + 0 \quad \text{then } a = -1 \]

\[ a \, x^2 + b \, x + c \]

\[ b = 19 \]

\[ \frac{-b}{2a} = \frac{-19}{2 \cdot (-1)} = \frac{19}{2} \quad [5] \]

This means when \[ x = \frac{19}{2} = 9.5 \quad \Rightarrow \quad y = 19 - 9.5 = 9.5 \]

the product of \( P = xy \) is maximized.

\[ \text{in part (a) we were only considering whole numbers, but} \]

\[ \text{the problem never said the numbers had to be whole numbers.} \]

\[ \text{The estimated answer was close and gave us an idea that} \]

\[ \text{the true result should be somewhere around 9 \neq 10.} \]