# Math 113

## Group Quiz 7 Solutions

(1) Rectangular building: 3 times as long as it is wide $\rightarrow$ $l = 3w$

Find a function that models the area $A$ in terms of width, $w$.

ie floorspace

**Known**

$l = 3w$ [8]

**want to know**

$A$ in terms of $w$

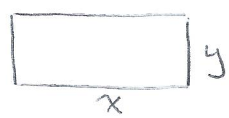
$A = l \cdot w$

substitute

$= (3w) \cdot w$ [10]

$\boxed{A = 3w^2}$ [2]

(2) Rectangle has area of $16\,m^2$. Find function to model perimeter $P$ in terms of length $x$ of one of its sides.




**Known**

$A = 16\ m^2 = x \cdot y$

$\downarrow$ (solve for y)

$y = \dfrac{16}{x}$

[8]

**Want to know**

$P$ in terms of $x$

$P = 2x + 2y$

substitute

$= 2x + 2\left(\dfrac{16}{x}\right)$ [10]

$= \boxed{2x + \dfrac{32}{x}}$ [2]

or

$= \boxed{2\left(\dfrac{x^2+16}{x}\right)}$

side note: The units for the Perimeter will be meters

(3) height of cylinder is four times the radius $\longrightarrow h=4r$

model Vol of cylinder in terms of r.


| Known | Want to know |
|---|---|
| $h=4r$ [8] | V in terms of r. |

substitute.

$$V=\pi r^2 \cdot h$$

$$=\pi r^2(4r) \quad [10]$$

$$\boxed{V=4\pi r^3} \quad [2]$$

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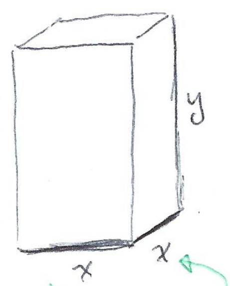
(4) box with volume $60 ft^3$ has a square base.

want. SA in terms of x (length of one side of base)




the problem tells us these are the same length.

| known | want to know |
|---|---|
| $V=60ft^3=x\cdot x\cdot y$ | SA in terms of x |

solve for y

$$y=\frac{60}{x^2}$$

[8]

notice: can be simplified

$$SA=2(x\cdot x+x\cdot y+x\cdot y)$$

$$=2(x^2+2xy) \quad [10]$$

substitute

$$=2\left(x^2+2x\left(\frac{60}{x^2}\right)\right)$$

$$=\boxed{2\left(x^2+\frac{120}{x}\right)} \quad [2]$$

or

$$=\boxed{2\left(\frac{x^3+120}{x}\right)}$$

5) 2 numbers whose sum is 19 ⟶ $x+y=19$

product is maximized ⟶ let $P = xy$

(want to maximize $P$)

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a)

| 1st #: $x$ | 2nd #: $y = 19-x$ | product: $P = xy$ |
|---|---|---|
| 1 | 18 | 18 |
| 2 | 17 | $2 \times 17 = 34$ |
| 3 | 16 | $3 \times 16 = 48$ |
| 4 | 15 | $4 \times 15 = 60$ |
| 5 | 14 | $5 \times 14 = 70$ |
| 6 | 13 | $6 \times 13 = 78$ |
| 7 | 12 | $7 \times 12 = 84$ |
| 8 | 11 | $8 \times 11 = 88$ |
| 9 | 10 | $9 \times 10 = 90$ |

[2]

Notice that if I continue the table, the numbers will repeat. $x$ & $y$ are chosen arbitrarily, so we can stop here. [1]

Estimated answer: $x = 9$, $y = 10$ will add to 19 & maximize the product $P = xy$.

b) model product in terms of $x$.

known: $x + y = 19$

$y = 19 - x$

[3]

want to know: $P$ in terms of $x$

$P = xy$

substitute

$= x(19-x)$

$\boxed{P = 19x - x^2}$ [4]

⑤ c) $P = 19x - x^2$ (from part b)

$p = -x^2 + 19x$ ← this is a quatratic with a negative coefficient in front

[1]

→ this means it will graph as a parabola opening downward

→ this means it will attain its maximum at the vertex $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

We only care about the x-value of the vertex since we want to know what x-value forces the product to be a maximum.

thus if $P = -1 \cdot x^2 + 19x + 0$ then $a = -1$

"$ax^2 + bx + c$" $b = 19$

so $\frac{-b}{2a} = \frac{-19}{2(-1)} = \frac{19}{2}$ [5]

This means when $\boxed{x = \frac{19}{2} = 9.5}$ & $\boxed{y = 19 - 9.5 = 9.5}$ the product of $P = xy$ is maximized. [3]

in part (a) we were only considering whole numbers, but the problem never said the numbers had to be whole numbers. The estimated answer was close & gave us an idea that the true result should be somewhere around 9 & 10. [1]