Math 113

Group Quiz 6 Solutions

(1) a) $f(x) = 2x^2 - 20x + 57$ (26 pt)

i) $f(x) = 2(x^2 - 10x + \underline{(-5)^2}) + 57 - 50$

$+50$

$f(x) = 2(x-5)^2 + 7$ [5] $\rightarrow$ vertex $(5, 7)$ [4]

ii) x-int: $y = 0$

$0 = 2(x-5)^2 + 7$

$-7 \quad\quad -7$

$-7 = 2(x-5)^2$ ← Never can happen since the right hand side is always positive.

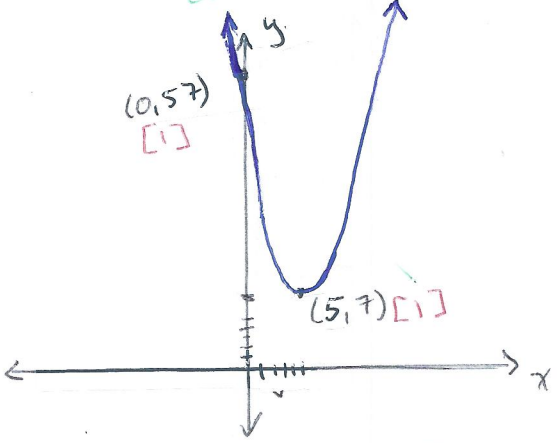
no x-intercept [4]

iii) y-int: $x = 0$

$f(0) = 2(0)^2 - 20(0) + 57 = 57 \rightarrow (0, 57)$ [4]

Use the original form of the quadratic to make this calculation easy.

iv)



[1 label axes]
[1 upward]
[1 symmetric]

v) The function f has a minimum [4]

at $x = 5$.

The graph opens upward.

(1) b) $f(x) = -x^2 + x + 2$

(26 pt) i) $f(x) = -1\left(x^2 - x + \left(\frac{-1}{2}\right)^2\right) + 2 + \frac{1}{4}$

$-\frac{1}{4}$

$f(x) = -1\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}$ [5] → vertex $\left(\frac{1}{2}, \frac{9}{4}\right)$ [4]

ii) x-int: set $y = 0$

$0 = -1\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}$

$-\frac{9}{4} = -1\left(x - \frac{1}{2}\right)^2$

$\frac{9}{4} = \left(x - \frac{1}{2}\right)^2$

$\pm \frac{3}{2} = x - \frac{1}{2}$

$\frac{1}{2} + \frac{3}{2} = x \rightarrow x = 2 \rightarrow (2, 0)$

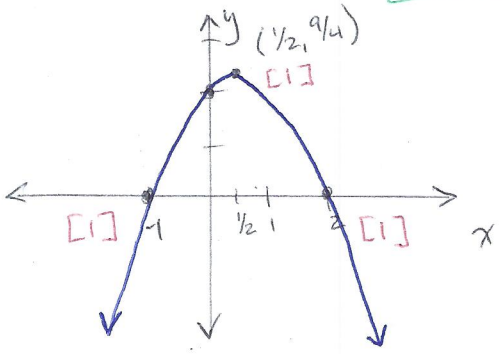
$\frac{1}{2} - \frac{3}{2} = x \rightarrow x = -1 \rightarrow (-1, 0)$

x-intercepts: $(2,0)$, $(-1,0)$ [4]

iii) y-int: set $x = 0$

$f(0) = 2 \rightarrow (0, 2)$ [4]

iv)



[1 label axes]

[1 downward]

v) The function f has a maximum [1] at $x = 1/2$ [2]

The graph opens downward. [1]

(2) (12pt)

vertex at $(-2, 8)$ $\longrightarrow$ $g(x) = a(x-(-2))^2 + 8$ [3]

this is by plugging into the standard form of $g(x) = a(x-h)^2 + k$ where (h, k) is the vertex.

x-int at $(-6, 0)$

$(2, 0)$

y-int at $(0, 6)$

opens downward

$$g(x) = a(x+2)^2 + 8 \quad [1]$$

using the point $(0,6)$ we get $g(0) = a(0+2)^2 + 8 = 6$

$$a(4) + 8 = 6$$

$$4a = -2$$

$$a = -\tfrac{1}{2} \quad [4]$$

thus $\boxed{g(x) = -\tfrac{1}{2}(x+2)^2 + 8}$ [4]

It is smart to check your other known facts.

☑ should open downward

☑ $(-6,0)$ is on the graph: $g(-6) = -\frac{1}{2}(-6+2)^2 + 8 \stackrel{?}{=} 0$

$-\frac{1}{2}(-4)^2 + 8 \stackrel{?}{=} 0$

$-\frac{16}{2} + 8 = 0$ ✓

☑ $(2,0)$ is on the graph: $g(2) = -\frac{1}{2}(2+2)^2 + 8 \stackrel{?}{=} 0$

$-\frac{1}{2}(16) + 8 = 0$ ✓

(3) Find all real solutions of the equation (12pt)

$$1 + \frac{2x}{(x+3)(x+4)} = \frac{2}{(x+3)} + \frac{4}{(x+4)}$$

LCD: $(x+3)(x+4)$ [4]

$$\frac{(x+3)(x+4)}{(x+3)(x+4)} + \frac{2x}{(x+3)(x+4)} = \frac{2(x+4)}{(x+3)(x+4)} + \frac{4(x+3)}{(x+3)(x+4)}$$

Since each term now has the common denominator we can work just with the numerators

$$(x+3)(x+4) + 2x = 2(x+4) + 4(x+3)$$

$$x^2 + 7x + 12 + 2x = 2x + 8 + 4x + 12$$

$$x^2 + 9x + 12 = 8x + 20$$

$$-8x \quad -20 \qquad -8x \quad -20$$

$$x^2 + x - 8 = 0$$

[4]

Complete the square (or use the quadratic formula)

$$\left(x^2 + x + \left(\tfrac{1}{2}\right)^2\right) - 8 - \tfrac{1}{4} = 0$$

$$\left(x + \tfrac{1}{2}\right)^2 - \frac{33}{4} = 0$$

$$+\tfrac{33}{4}$$

$$\sqrt{\left(x + \tfrac{1}{2}\right)^2} = \sqrt{\frac{33}{4}}$$

$$\left(x + \tfrac{1}{2}\right) = \pm\sqrt{\frac{33}{4}} = \pm\frac{\sqrt{33}}{2}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{33}}{2}$$

$$\boxed{x = \frac{-1 \pm \sqrt{33}}{2}}$$

[4]

(4) (12 pt)

vertex: $(1, -2)$

passes through $(4, 16)$

$f(x) = a(x-1)^2 - 2$ [4] enter your known information into the standard form $f(x) = a(x-h)^2 + k$

$f(4) = a(4-1)^2 - 2 = 16$ use the given point to solve for a.

$a \cdot 9 - 2 = 16$

$9a = 18$

$a = 2$ [4]

$f(x) = 2(x-1)^2 - 2$ [4]

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(5) (12 pt)

$P(x) = -0.001x^2 + 3x - 1800$

$P(x) = -0.001(x^2 - 3000x + (1500)^2) - 1800 + 2250$ [4]

-2250

$P(x) = -0.001(x - 1500)^2 + 450$

↳ vertex $(1500, 450)$ [4]

His maximum profit per day is \$450 & he must sell 1500 cans of soft-drink to get maximum profit.

[4 complete sentence]