

Math 113
Group Quiz 5 Solutions

① a) $f(1) = 3$

[4] $f(3) = 3$

$f(5) = 1$

$f(6) =$ Does not exist or Undefined

b) where is $f(x) > 3$?

[4] $(1, 3)$

notice we exclude the end points of this interval because the question is using $>$ rather than \geq

c) where does $f(x) = 1$?

[6] $[0, 0] \cup (3, 5]$

d) net change of f between $x=3$ and $x=5$:

[5] $f(5) - f(3) = 1 - 3 = -2$

e) slope of secant line between $x=0$, $x=2$:

[6] $\frac{f(2) - f(0)}{2 - 0} = \frac{5 - 1}{2} = \frac{4}{2} = 2$

f) any 2 points which are level horizontally are acceptable end points

[5] eg: $[0, 5]$, $[1, 4]$, $[4, 5]$

$[1, 3]$...

② a) Avg ROC of soup temp over $t=0$ to $t=20$.

[8 (1 for unit)]

$$AROC_{[0,20]} = \frac{T(20) - T(0)}{20 - 0} = \frac{119 - 200}{20} = \frac{-81}{20} = \boxed{-4.05 \frac{^{\circ}\text{F}}{\text{min}}}$$

This result means that over the first twenty minutes the soup loses 4.05 degrees F of heat per minute, on average.

b) Avg ROC of soup temp over $t=20$ to $t=40$

[8 (1 for unit)]

$$AROC_{[20,40]} = \frac{T(40) - T(20)}{40 - 20} = \frac{89 - 119}{20} = \frac{-30}{20} = -\frac{3}{2} = \boxed{-1.5 \frac{^{\circ}\text{F}}{\text{min}}}$$

This result means that over the second 20 minutes, the soup loses 1.5 degrees F of heat per minute, on average.

c) The soup cooled more quickly in the first 20 minute interval, because it lost more heat per minute on average.

[4]

③ a) $f(x) = x^2 + 2x$ $x_1 = -1, x_2 = 4$

$$AROC_{[x_1, x_2]} = \frac{f(4) - f(-1)}{4 - (-1)} = \frac{(16 + 8) - (1 - 2)}{5} = \frac{24 + 1}{5} = \frac{25}{5} = \boxed{5}$$

[10]

b) $f(x) = \frac{2}{x}$ $x_1 = a$ $x_2 = a+h$ LCD: $(a+h)(a)$

$$AROC_{[x_1, x_2]} = \frac{f(a+h) - f(a)}{a+h - a} = \frac{\frac{2}{a+h} - \frac{2}{a}}{h} = \frac{\frac{2a - 2(a+h)}{a(a+h)}}{h}$$

$$= \frac{2a - 2a - 2h}{a(a+h)h} = \frac{-2h}{a(a+h)h} = \boxed{\frac{-2}{a(a+h)}}$$

[10]

④ a) Domain of h : $D_h \equiv [-2, 9]$

[4] b) Range of h : $R_h \equiv \{y \mid -5 \leq y \leq 6\}$

[6] c) h is increasing on $[1, 2] \cup [3, 6) \cup [8, 9]$

[6] d) h is decreasing on $[-2, 1] \cup [2, 3] \cup [6, 8]$

[5] e) " h has a local minimum value of -5, attained at $x=1$ and $x=3$ "

[5] " h has a local maximum value of 6, attained at $x=6$ "