Math 113
Group Quiz 2 Solutions

(1) (a) (10pt) $(-8,0)$, $(0,6)$

$m = \frac{6-0}{0-(-8)} = \frac{6}{8} = \frac{3}{4}$ [5] $\longrightarrow$ $\boxed{y = \frac{3}{4}x + 6}$ [5]

(1) find slope
(2) we know the y-intercept is $(0,6)$, so the "b" of $y=mx+b$ must be 6.

(b) (10pt) $(-2,5)$ $(-1,-3)$

$m = \frac{-3-5}{-1-(-2)} = \frac{-8}{1} = -8$ [4] $\longrightarrow$

$y - y_1 = m(x - x_1)$

$y - 5 = -8(x - (-2))$ [2]

$y - 5 = -8(x+2)$

$y - 5 = -8x - 16$

$+5 \qquad +5$

$\boxed{y = -8x - 11}$ [4]

(1) find slope
(2) use point-slope form with either point
(3) change equation into slope-int. form.

(c) (10pt) $(-1,3)$ & perpendicular to $3x + 6y = 9$ $\longrightarrow$ $6y = -3x + 9$

$y = \frac{-3}{6}x + \frac{9}{6}$

$y = -\frac{1}{2}x + \frac{3}{2}$

$\hookrightarrow m_1 = -\frac{1}{2}$ [3]

(1) find the slope of the line $3x + 6y = 9$
(2) recall that if 2 lines are perpendicular the slopes are related by $m_2 = -\frac{1}{m_1}$
(3) use point-slope form to get your new line
(4) change equation to slope-int. form

$m_2 = 2$ & $(-1,3)$

$y - 3 = 2(x - (-1))$ [4]

$y - 3 = 2x + 2$

$\boxed{y = 2x + 5}$ [3]

(2) (20 points) \$2200 to make 100 chairs $\longrightarrow (100, 2200)$

\$4800 to make 300 chairs $\longrightarrow (300, 4800)$

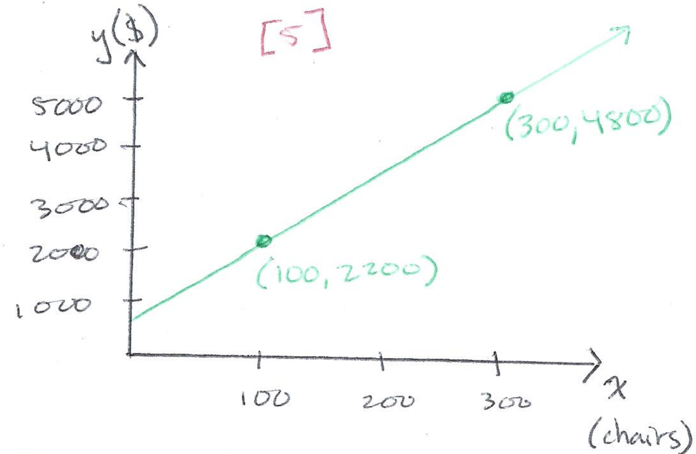
a)
$$m = \frac{4800-2200}{300-100} = \frac{2600}{200} = 13 \ \frac{\$}{\text{chair}}$$

note the units help answer part b.

$$y - 2200 = 13(x - 100)$$

$$y - 2200 = 13x - 1300$$
$$+2200 \qquad\qquad +2200$$
$$\boxed{y = 13x + 900}$$

[5]



b) the slope, m = 13, means it costs 13 dollars per each additional chair produced. [5]

Note: the independent variable is the # of chairs produced in a day.
the dependent variable is the cost, which depends on the # of chairs produced.

c) The y-intercept, (0, 900) means that even if no chairs are produced, it still costs \$900 to run the factory for one day. [5]

(3) a) $f(x) = \dfrac{x+2}{x^2-1}$ (10 pt)

- the numerator can take any number for input, so it causes no limitations to the domain.
- the denominator cannot $=0$ (we can't divide by zero!!)

$x^2-1 \neq 0$ ← Factor the difference of squares.

$(x-1)(x+1) \neq 0$ [1]

2 cases

$x-1 \neq 0$

$x \neq 1$ [4]

$x+1 \neq 0$

$x \neq -1$

$\boxed{D(f) = (-\infty,-1)\cup(-1,1)\cup(1,\infty)}$

$\boxed{= \{x \mid x \neq -1, 1\}}$ [5]

b) $myf(x) = \sqrt{x-5}$ (10 pt)

- We cannot take the square root of a negative number, so we must force $x-5$ to be at least 0.

$x-5 \geq 0$ [5]

$x \geq 5 \longrightarrow \boxed{D(myf) = [5,\infty)} = \boxed{\{x \mid x \geq 5\}}$ [5]

c) $urf(x) = \dfrac{\sqrt{2+x}}{3-x}$ (10 pt)

I. cannot take square root of negative, so $2+x \geq 0$

II cannot divide by zero, so $3-x \neq 0$

I $2+x \geq 0$

$x \geq -2$

II $3-x \neq 0$

$x \neq 3$ [5]

$\boxed{D(urf) = [-2,3)\cup(3,\infty) = \{x \mid x \geq -2 \text{ ; } x \neq 3\}}$ [5]

4) difference quotient: $\frac{f(a+h)-f(a)}{h}$, $h \neq 0$

a) $f(x) = 3 - 5x - 4x^2$

(10 pts) $f(a) = 3 - 5a - 4a^2$

it is helpful to write this part first before entering it all into the difference quotient.

$f(a+h) = 3 - 5(a+h) - 4(a+h)^2$

$= 3 - 5a - 5h - 4[a^2 + 2ah + h^2]$

$= 3 - 5a - 5h - 4a^2 - 8ah - 4h^2$

$$\frac{f(a+h)-f(a)}{h} = \frac{[3-5a-5h-4a^2-8ah-4h^2]-[3-5a-4a^2]}{h}$$

[5] be sure to distribute the negative!!

$$= \frac{\cancel{3}\cancel{-5a}-5h\cancel{-4a^2}-8ah-4h^2\cancel{-3}\cancel{+5a}\cancel{+4a^2}}{h}$$

$$= \frac{-5h-8ah-4h^2}{h}$$

-factor an h out of every term

$$= \frac{\cancel{h}(-5-8a-4h)}{\cancel{h}}$$

-cancel the h in the numerator & denominator.

$$= \boxed{-5-8a-4h}$$ [5]

b) $f(x) = \frac{2x}{x-1}$

(10 pt)

$f(a) = \frac{2a}{a-1}$

$f(a+h) = \frac{2(a+h)}{a+h-1}$

$= \frac{2a+2h}{a+h-1}$

$$\frac{f(a+h)-f(a)}{h} = \frac{1}{h}\left[\frac{2a+2h}{a+h-1} - \frac{2a}{a-1}\right]$$ [5]

can keep $\frac{1}{h}$ out front to save space & avoid long division lines.

$$= \frac{1}{h}\left[\frac{(2a+2h)}{(a+h-1)}\cdot\frac{(a-1)}{(a-1)} - \frac{(2a)}{(a-1)}\cdot\frac{(a+h-1)}{(a+h-1)}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{2a^2}\cancel{-2a}\cancel{+2ah}-2h\cancel{-2a^2}\cancel{-2ah}\cancel{+2a}}{(a-1)(a+h-1)}\right]$$

$$= \frac{1}{\cancel{h}}\left[\frac{-2\cancel{h}}{(a-1)(a+h-1)}\right]$$

$$= \boxed{\frac{-2}{(a-1)(a+h-1)}}$$ [5]