Math 113

Group Quiz 21 Solutions

1. (28 pts) Expand [4 each]

a) $\log_2(x(x-1)) = \boxed{\log_2(x) + \log_2(x-1)}$

b) $\log_4 \frac{y}{y+3} = \boxed{\log_4 y - \log_4(y+3)}$

c) $\log_5 \sqrt[3]{x^2+1} = \log_5 (x^2+1)^{1/3} = \boxed{\frac{1}{3}\log_5 (x^2+1)}$

d) $\log_5 \sqrt{2a^4+1} = \log_5 (2a^4+1)^{1/2} = \boxed{\frac{1}{2}\log_5 (2a^4+1)}$

e) $\log\left(\frac{x^3y^4}{z^6}\right) = \log x^3 + \log y^4 - \log z^6$

$= \boxed{3\log x + 4\log y - 6\log z}$

f) $\log_2\left(\frac{x(x^2+1)}{\sqrt{x^2-1}}\right) = \log_2 x + \log_2(x^2+1) - \log_2 (x^2-1)^{1/2}$

$= \log_2 x + \log_2(x^2+1) - \frac{1}{2}\log_2((x-1)(x+1))$

$= \log_2 x + \log_2(x^2+1) - \frac{1}{2}\left[\log_2(x-1) + \log_2(x+1)\right]$

$= \boxed{\log_2 x + \log_2(x^2+1) - \frac{1}{2}\log_2(x-1) - \frac{1}{2}\log_2(x+1)}$

g) $\ln\left(x\sqrt{\frac{y}{z}}\right) = \ln x + \ln\left(\frac{y}{z}\right)^{1/2}$

$= \ln x + \frac{1}{2}\ln\left(\frac{y}{z}\right)$

$= \ln x + \frac{1}{2}\left[\ln y - \ln z\right]$

$= \boxed{\ln x + \frac{1}{2}\ln y - \frac{1}{2}\ln z}$

(2) Combine

(12 pts) [4 each]

a) $3\log_2 A + 5\log_2 B - 2\log_2 C$

$= \log_2 A^3 + \log_2 B^5 - \log_2 C^2$

$= \boxed{\log_2\left(\frac{A^3B^5}{C^2}\right)}$

b) $4\log x - \frac{1}{3}\log(x^2+1) + 2\log(x-1)$

$= \log x^4 - \log\sqrt[3]{x^2+1} + \log(x-1)^2$

$= \boxed{\log\left(\frac{x^4(x-1)^2}{\sqrt[3]{x^2+1}}\right)}$

c) $2(\log_5 x + 2\log_5 y - 3\log_5 z)$

$= 2(\log_5 x + \log_5 y^2 - \log_5 z^3)$

$= 2\left(\log_5 \frac{xy^2}{z^3}\right)$

$= \log_5\left(\frac{xy^2}{z^3}\right)^2$

$= \boxed{\log_5\left(\frac{x^2y^4}{z^6}\right)}$

2]

③ True or False

(30 pt)

[3 each]

a) $\log\left(\frac{x}{y}\right) = \frac{\log x}{\log y}$ $\boxed{\text{False}}$: Counter Example, let $x = 100$, $y = 10$

$\log\left(\frac{100}{10}\right) = \log 10 = 1$

$\frac{\log 100}{\log 10} = \frac{2}{1} = 2$

b) $\log_2(x-y) = \log_2 x - \log_2 y$ $\boxed{\text{False}}$: Counter example, let $x = 6$, $y = 4$

$\log_2(6-4) = \log_2 2 = 1$

$\log_2 6 - \log_2 4 = 2.58 - 2 = 0.58$

c) $\log_5\left(\frac{a}{b^2}\right) = \log_5 a - 2\log_5 b$ $\boxed{\text{True}}$

d) $\log 2^z = z \log 2$ $\boxed{\text{True}}$

e) $(\log P)(\log Q) = \log P + \log Q$ $\boxed{\text{False}}$: Counterexample let $P = 100$, $Q = 10$

$(\log 100)(\log 10) = 2 \cdot 1 = 2$

$(\log 100) + \log 10 = 2 + 1 = 3$

f) $\frac{\log a}{\log b} = \log a - \log b$ $\boxed{\text{False}}$: Counter example: $a = 100$, $b = 100$

$\frac{\log 100}{\log 100} = 1$

$\log 100 - \log 100 = 0$

g) $(\log_2 7)^x = x \cdot \log_2 7$ $\boxed{\text{False}}$: let $x = 2$

$(\log_2 7)^2 = 1.87$

$2(\log_2 7) = 2.73$

3

③ h) $\log_a a^a = a^a$

$\boxed{\text{False:}}$ Counterexample

let $a = 2$

$\log_2 2^2 = 2$

$2^2 = 4$

i) $\log(x-y) = \dfrac{\log x}{\log y}$

$\boxed{\text{False:}}$ Counter example

let $x = 20$

$y = 10$

$\log(20-10) = \log 10 = 1$

$\dfrac{\log 20}{\log 10} = \dfrac{1.3}{1} = 1.3$

j) $-\ln\left(\dfrac{1}{A}\right) = \ln A$

$\boxed{\text{True}}$

because $-1 \cdot \ln\left(\frac{1}{A}\right) = -1 \cdot \ln(A^{-1})$

$= \ln(A^{-1})^{-1}$

$= \ln(A)$

(4) (10 pts) in cheeses, hydrogen ion concentration ranges from

4.0×10^{-7} M to 1.6×10^{-5}

$$pH = -\log[4 \times 10^{-7}]$$
$$= -[\log 4 + \log 10^{-7}]$$
$$= -[\log 4 - 7]$$
$$= -\log 4 + 7$$
$$= 6.40 \quad [5]$$

$$pH = -\log[1.6 \times 10^{-5}]$$
$$= -[\log 1.6 + \log 10^{-5}]$$
$$= -[\log 1.6 + -5]$$
$$= -\log 1.6 + 5$$
$$= 4.80$$
[5]

$\boxed{4.80 \le pH \le 6.40}$

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(5) pH in wine varies from 2.8 to 3.8

pH = 2.8

let $H^+ = x$

$$2.8 = -\log[H^+]$$
$$2.8 = -\log x$$
$$-2.8 = \log x$$
$$x = 10^{-2.8}$$
$$x = 0.00158$$
$$x = 1.58 \times 10^{-3}$$
[5]

pH = 3.8

let $H^+ = x$

$$3.8 = -\log[H^+]$$
$$3.8 = -\log x$$
$$-3.8 = \log x$$
$$x = 10^{-3.8}$$
$$x = 0.000158$$
$$x = 1.58 \times 10^{-4}$$
[5]

$\boxed{H^+ \text{ varies from } 1.58 \times 10^{-4} \text{ M to } 1.58 \times 10^{-3} \text{ M.}}$

6 $M_{CA} = 6.8$

(10 pts)

$M_J = 7.2$

want to know $\dfrac{I_J}{I_{CA}}$.

since $\dfrac{I_J}{I_{CA}} = \dfrac{I_J}{S} \cdot \dfrac{S}{I_{CA}} = \dfrac{\frac{I_J}{S}}{\frac{I_{CA}}{S}}$ [5] setup

$$\log \frac{I_J}{I_{CA}} = \log\left(\frac{\frac{I_J}{S}}{\frac{I_{CA}}{S}}\right) = \log\left(\frac{I_J}{S}\right) - \log\left(\frac{I_{CA}}{S}\right)$$

$$= M_J - M_{CA}$$

$$= 7.2 - 6.8$$

$$\log \frac{I_J}{I_{CA}} = 0.4$$

$$\frac{I_J}{I_{CA}} = 10^{0.4} = 2.51$$

Thus the earthquake in Japan was 2.51 times more intense than the earthquake in California.

[5]