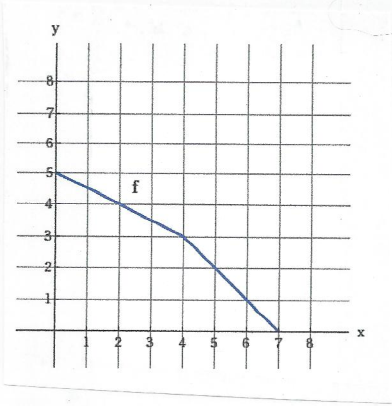
Math 113
Group Quiz 20 Solutions

1 (6pt)



a) $\boxed{f^{-1}(2) = 5}$ because $f(5) = 2$ [2]

b) $\boxed{f^{-1}(5) = 0}$ because $f(0) = 5$ [2]

c) $\boxed{f^{-1}(6) \text{ Does Not Exist}}$ because [2]
6 is not in the range of f,
thus, 6 is not in the domain of f^{-1}.

2

a) $f(x) = \dfrac{x-5}{3x+4}$ let $x = \dfrac{y-5}{3y+4}$

$$3yx + 4x = y - 5$$

$$(3x-1)y = -4x - 5$$

$$\boxed{f^{-1}(x) = y = \frac{-4x-5}{3x-1}}$$

[6]

check

$$f \circ f^{-1}(x) = f\left(\frac{-4x-5}{3x-1}\right)$$

$$= \frac{\frac{-4x-5}{3x-1} - 5}{3\left(\frac{-4x-5}{3x-1}\right) + 4}$$

$$= \frac{\left(\frac{-4x-5}{3x-1} - 5\right)(3x-1)}{\left(\frac{-12x-15}{3x-1} + 4\right)(3x-1)}$$

$$= \frac{-4x \cancel{-5} - 15x \cancel{+5}}{\cancel{-12x} - 15 + \cancel{12x} - 4}$$

$$= \frac{-19x}{-19}$$

$$f \circ f^{-1}(x) = x \checkmark$$

[2]

check:

$$f^{-1} \circ f(x) = f^{-1}\left(\frac{x-5}{3x+4}\right)$$

$$= \frac{-4\left(\frac{x-5}{3x+4}\right) - 5}{3\left(\frac{x-5}{3x+4}\right) - 1}$$

$$= \frac{\left(\frac{-4x+20}{3x+4} - 5\right)(3x+4)}{\left(\frac{3x-15}{3x+4} - 1\right)(3x+4)}$$

$$= \frac{-4x + \cancel{20} - 15x \cancel{-20}}{\cancel{3x} - 15 - \cancel{3x} - 4}$$

$$= \frac{-19x}{-19}$$

$$f^{-1} \circ f(x) = x \checkmark$$

[2]

2 b) $f(x) = 4 - x^2$, $x \geq 0 \rightarrow D_f \equiv [0,\infty) \rightarrow R_{f^{-1}} \equiv [0,\infty)$

this requirement is needed so that f is a one-to-one function satisfying the horizontal line test.

[1]

let $x = 4 - y^2$

$y^2 = 4 - x$

$y = \cancel{\pm}\sqrt{4-x} \rightarrow \boxed{f^{-1}(x) = \sqrt{4-x}}$ [5]

since $D_f \equiv [0,\infty)$, $R_{f^{-1}} \equiv [0,\infty)$ so these new y values must be nonnegative.

check

$f^{-1} \circ f(x) = f^{-1}(4-x^2)$

$= +\sqrt{4-(4-x^2)}$

$= \sqrt{4-4+x^2}$

$= \sqrt{x^2}$

$f^{-1} \circ f(x) = x$ ✓ since x was given as $x \geq 0$

[2]

check

$f \circ f^{-1}(x) = f(\sqrt{4-x})$

$= 4 - (\sqrt{4-x})^2$

$= 4 - (4-x)$

$= 4 - 4 + x$

$f \circ f^{-1}(x) = x$ ✓

[2]

② c) $f(x) = 4 + \sqrt[3]{x}$

let $x = 4 + \sqrt[3]{y}$

$x - 4 = \sqrt[3]{y}$

$(x-4)^3 = y$

[6] $\boxed{f^{-1}(x) = (x-4)^3}$

check

$f^{-1} \circ f(x) = f^{-1}(4 + \sqrt[3]{x})$

$= ((4 + \sqrt[3]{x}) - 4)^3$

$= (4 + \sqrt[3]{x} - 4)^3$

$= (\sqrt[3]{x})^3$

$f^{-1} \circ f(x) \quad = x$

[2]

check

$f \circ f^{-1}(x) = f((x-4)^3)$

$= 4 + \sqrt[3]{(x-4)^3}$

$= 4 + (x-4)$

$f \circ f^{-1}(x) = x$

[2]

③ a) Express $\ln(x-1) = -4$ in exponential form: $\boxed{e^{-4} = x - 1}$ [8]

b) $e^{x+1} = 0.5 \longrightarrow$ log form: $\boxed{\ln(0.5) = x + 1}$ [8]

c) Solve $3 - \log_3(5x+7) = \cancel{4}$ $+\log_3(5x+7)$ $\cancel{-4}$

$+\log_3(5x+7)$

-4

$3 - 4 = \log_3(5x+7)$

$-1 = \log_3(5x+7)$

translate to exponential form

or

put $3^{-1} = 3^{\log_3(5x+7)}$

& simplify using property 4.

$3^{-1} = 5x + 7$

$-7 \qquad -7$

$\frac{1}{3} - 7 = 5x$

$\overline{5} \qquad \overline{5}$

$\boxed{x = \frac{\frac{1}{3} - 7}{5} = \frac{-4}{3} = -1.33}$ [8]

(4) $\log_8(0.25) - \log_6 1 + 10^{\log_{10} 87} + 2\ln\left(\frac{1}{e}\right) - e^{\ln \pi^2}$
(a) (b) (c) (d) (e)

$$= \underbrace{-\frac{2}{3} - 0 + 87 - 2 - \pi^2}$$

$$= 85 - \frac{2}{3} - \pi^2$$

$$= \boxed{84\tfrac{1}{3} - \pi^2} \quad [3]$$

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let (a) $\log_8(0.25) = x$

$\log_8\left(\frac{1}{4}\right) = x \xrightarrow[\text{exp form}]{} 8^x = \frac{1}{4}$

$(2^3)^x = \frac{1}{2^2}$

$2^{3x} = 2^{-2} \longrightarrow 3x = -2$ [1]

$x = -\frac{2}{3} \longrightarrow \boxed{\log_8(0.25) = \frac{-2}{3}}$

(b) $\boxed{\log_6 1 = 0}$ because $\log_a 1 = 0$ for all a because $a^0 = 1$ [1]

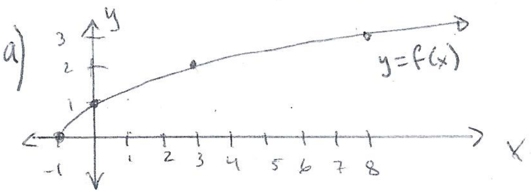
(c) $\boxed{10^{\log_{10} 87} = 87}$ because $a^{\log_a x} = x$ [1]

(d) $2\ln\left(\frac{1}{e}\right) = 2\underbrace{\ln e^{-1}}_{=-1 \text{ because } \ln e^x = x} = 2(-1) = -2 \longrightarrow \boxed{2\ln\left(\frac{1}{e}\right) = -2}$ [1]

(e) $\boxed{e^{\ln \pi^2} = \pi^2}$ because $e^{\ln x} = x$ [1]

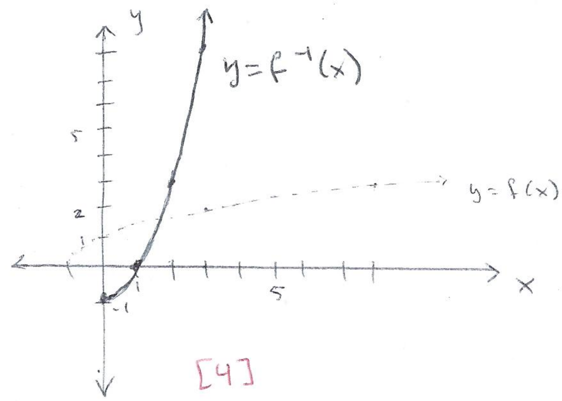
5) $f(x) = \sqrt{x+1}$

a)



[4]

b)



[4]

c) let $x = \sqrt{y+1}$

$x^2 = y+1$

$x^2 - 1 = y$

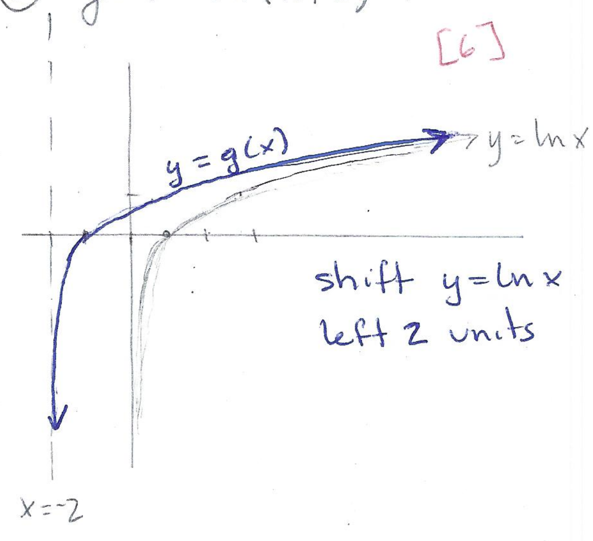
[4]

$$\boxed{f^{-1}(x) = x^2 - 1 \quad \text{where} \quad x \geq 0}$$

this is needed so that f^{-1} is also one-to-one & so that $D_{f^{-1}} = R_f$.

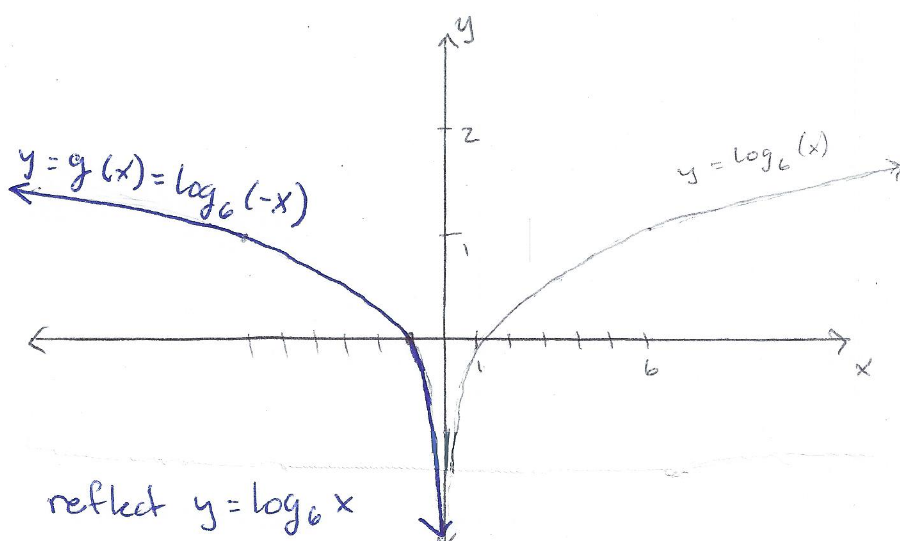
6) a) $g(x) = \ln(x+2)$

[6]



shift $y = \ln x$ left 2 units

b) [6] $g(x) = \log_6(-x)$



reflect $y = \log_6 x$ about the y-axis

(7) $f(x) = 3 + \log_5(8-2x)$

(8 pt)

note Domain of $\log_5 x$ is $(0,\infty)$

thus $8-2x > 0$

$8 > 2x$

$4 > x$

$x < 4 \longrightarrow \boxed{D_f \equiv (-\infty, 4)}$ [2]

note Range of $\log_5 x$ is $(-\infty,\infty)$

the changes do not affect this range! $\longrightarrow \boxed{R_f \equiv (-\infty,\infty)}$ [2]

note: the V.A of $\log_5(x)$ is $x=0$

thus the V.A of $\log_5(8-2x)$ is where $8-2x=0$

$8=2x$

$x=4$

Since $3+\log_5(8-2x)$ only moves the graph up 3 units,

the $\boxed{\text{V.A remains at } x=4}$ [4]