Math 113

Group Quiz 19 Solutions

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1) [6 each]

a) $g(x) = 2^{x-1.75}$

$g\left(-\frac{5}{4}\right) = 2^{\left(-\frac{5}{4}-1.75\right)} = 2^{(-1.25-1.75)} = 2^{(-3)} = \frac{1}{2^3} = \frac{1}{8} = 0.125$

$\boxed{g\left(-\frac{5}{4}\right) = 0.125}$

b) $h(x) = \frac{1}{3^{2x}}$

$h(-\sqrt{2}) = \frac{1}{3^{2(-\sqrt{2})}} \approx \boxed{22.362}$

c) $k(x) = 165e^{2x} + 3$

$k(-0.95) = 165e^{2(-0.95)} + 3$

$\approx \boxed{27.679}$

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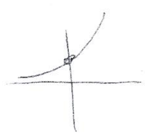
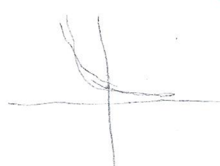
2) [3 each]

a) $f(x) = 2^x$



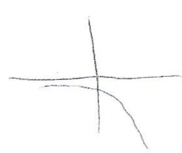
$\boxed{\text{III}}$

b) $f(x) = 2^{-x}$



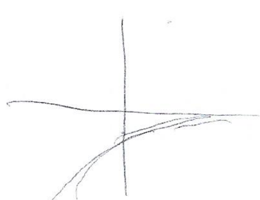
$\boxed{\text{I}}$

c) $f(x) = -2^x$



$\boxed{\text{II}}$

d) $f(x) = -2^{-x}$

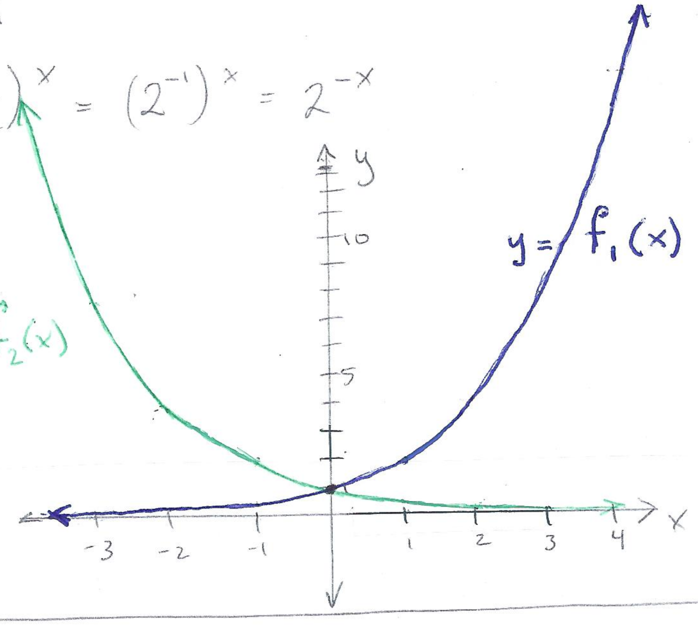


$\boxed{\text{IV}}$

(3) $f_1(x) = 2^x$

(12 pt) $f_2(x) = \left(\frac{1}{2}\right)^x = (2^{-1})^x = 2^{-x}$

[6 each]



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(4) Given: doubling time = 6 yrs

$n(30) = 100{,}000$

Want: formula for squirrel population

work backward:

t	n(t)
30	100,000
24	50,000
18	25,000
12	12,500
6	6,250
0	3,125

6 yrs earlier; $\frac{1}{2}$

(a) initial population 3125 squirrels [5]

$$\boxed{n(t) = 3125 \cdot 2^{t/6}}$$

b) # Squirrels 10 years from now = $n(40) = 3125 \cdot 2^{40/6} = 317{,}480.21\ldots$

$\approx$ $\boxed{317{,}480 \text{ squirrels}}$

[5]

("now" is year 30 so 10 years from now is year 40.)

(4) c) $n(t) = 3125 \cdot 2^{t/6}$

want to know # of years from now # squirrels reaches 200,000

$200{,}000 = 3125 \cdot 2^{t/6}$ solve for t.

$64 = 2^{t/6}$

$(2^6) = 2^{t/6}$

$6 = t/6$

$t = 36$ ← in year 36, [5]

ie $\boxed{\text{6 years from now}}$,

the population will reach 200,000 squirrels.

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(5) Given: ½ life of cesium-137 is 30 years → $h = 30$

initial amount: 10 g

a) $\boxed{m(t) = 10 \cdot 2^{-t/30}}$ [5]

half-life

initial amount

b) $m(80) = 10 \cdot 2^{-80/30} = \boxed{1.5749 \text{ g}}$ remains after 80 years.

c) $2 = 10 \cdot 2^{-t/30}$ solve for t [5]

$.2 = 2^{-t/30}$

$\ln(.2) = -\frac{t}{30} \cdot \ln(2)$

$t = -\frac{30 \cdot \ln(0.2)}{\ln(2)} = 69.66 \approx \boxed{70 \text{ years}}$ [5] → 2 grams remain after 70 years.

(6)

250 mg $\rightarrow$ initial amount $\rightarrow$

$(t, m(t))$
$(0, 250)$
$(48, 200)$

200 mg in 48 hr

units

initial amount

$m(t) = 250 \cdot 2^{-t/h}$ want to find h. [2 setup]

plug in $(48, 200)$ $\rightarrow$

$\frac{200}{250} = \frac{250 \cdot 2^{-48/h}}{250}$ [2 plug in]

$\frac{200}{250} = 2^{-48/h}$

$\ln(0.8) = \ln\left(2^{-48/h}\right)$

$\frac{h}{\ln(0.8)} \cdot \ln(0.8) = \frac{-48}{h} \cdot \ln(2) \cdot \frac{\cancel{h}}{\ln(0.8)}$

[8]

$h = \frac{-48 \ln(2)}{\ln(0.8)} \approx \boxed{149.1 \text{ hours} = \text{half life}}$

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(7) $P(t) = \frac{d}{1 + ke^{-ct}}$ c, d, k positive constants.

$d = 1200$
$k = 11$
$c = 0.2$

$P(t) = \frac{1200}{1 + 11e^{-0.2t}}$ $t \rightarrow$ years.

a) $P(0) = \frac{1200}{1 + 11e^{0}} = \frac{1200}{1 + 11} = \boxed{100 \text{ fish}}$ [4]

originally in the pond

b) $P(10) = \frac{1200}{1 + 11e^{-0.2(10)}} = \frac{1200}{1 + 11e^{-2}} = 482.18 \approx \boxed{482 \text{ fish after 10 years}}$ [1 each]

[1 units]

$P(20) = \frac{1200}{1 + 11e^{-0.2(20)}} = \frac{1200}{1 + 11e^{-4}} = 998.77 \approx \boxed{999 \text{ fish after 20 years}}$

$P(30) = \frac{1200}{1 + 11e^{-0.2(30)}} = \frac{1200}{1 + 11e^{-6}} = 1168.15 \approx \boxed{1168 \text{ fish after 30 years}}$

7 c) $P(t) = \dfrac{1200}{1+11e^{-0.2t}}$

as $t \to \infty$, $e^{-0.2t} \to 0$

the fish population approaches 1200 fish as time goes on

[2]

thus $11e^{-0.2t} \to 0$

thus $1+11e^{-0.2t} \to 1$

thus $\dfrac{1200}{1+11e^{-0.2t}} \to 1200$

[2 reasoning]

d) when will $P(t) = 900$?

$\left(\dfrac{1+11e^{-0.2t}}{900}\right) 900 = \dfrac{1200}{1+11e^{-0.2t}} \left(\dfrac{1+11e^{-0.2t}}{900}\right)$ solve for t.

[1 for setup]

$1+11e^{-0.2t} = \dfrac{1200}{900} = \dfrac{4}{3}$ -1 -1

$\dfrac{11e^{-0.2t}}{11} = \dfrac{1}{3} \cdot \dfrac{1}{11}$

$\ln\left(e^{-0.2t}\right) = \ln\left(\dfrac{1}{33}\right)$

$(-0.2t)\ln e = \ln\left(\dfrac{1}{33}\right)$

$\left(\dfrac{1}{-0.2}\right) -0.2t = \ln\left(\dfrac{1}{33}\right)\left(\dfrac{1}{-0.2}\right)$

[3]

$t = \dfrac{\ln\left(\frac{1}{33}\right)}{-0.2} = \boxed{17.5 \text{ years}}$ for the population to reach 900 fish.