Math 113
Group Quiz 18 Solutions

① When does a rational function have a slant asymptote?

(5pt) A rational function has a slant asymptote, or oblique asymptote whenever the degree of the numerator is strictly greater than the degree of the denominator.

$$\deg(N(x)) > \deg(D(x))$$

② Is it okay for parts of a rational function to cross through a slant asymptote? Why or why not?

(5pt) Yes! The rational function only nears the slant asymptotes at the far ends near ∞ and $-\infty$. What happens in the middle of the graph has nothing to do with the line or curve of the slant asymptote.

③ a) $r(x) = \dfrac{x^2}{x-2}$

x-int: set $y = r(x) = 0$ → when $x^2 = 0$, $x = 0$ → (0,0) x-int, y-int

y-int → (0,0) x-int, y-int

VA: set denominator = 0 → VA @ $x = 2$

[5]

HA: no HA since deg(N) > deg(D)

↳ Long division:

$$x-2 \;\overline{)\; x^2 + 0x + 0}$$

$x^2 - 2x$ ↓

$2x + 0$

$2x - 4$

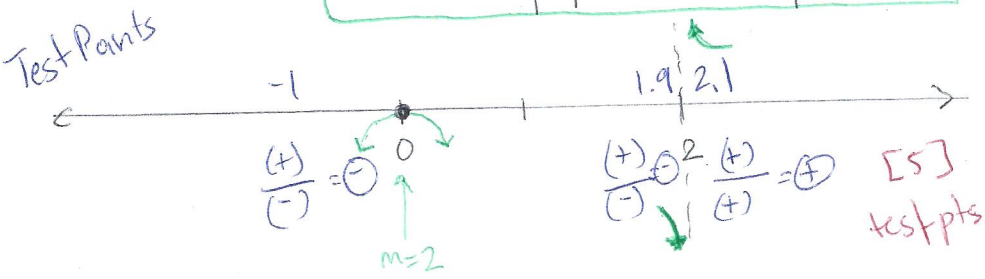
4

$x + 2 + \dfrac{4}{x-2}$

Since $\dfrac{x^2}{x-2} = x + 2 + \dfrac{4}{x-2}$ and as $x \to \infty$, $\dfrac{4}{x-2} \to 0$,

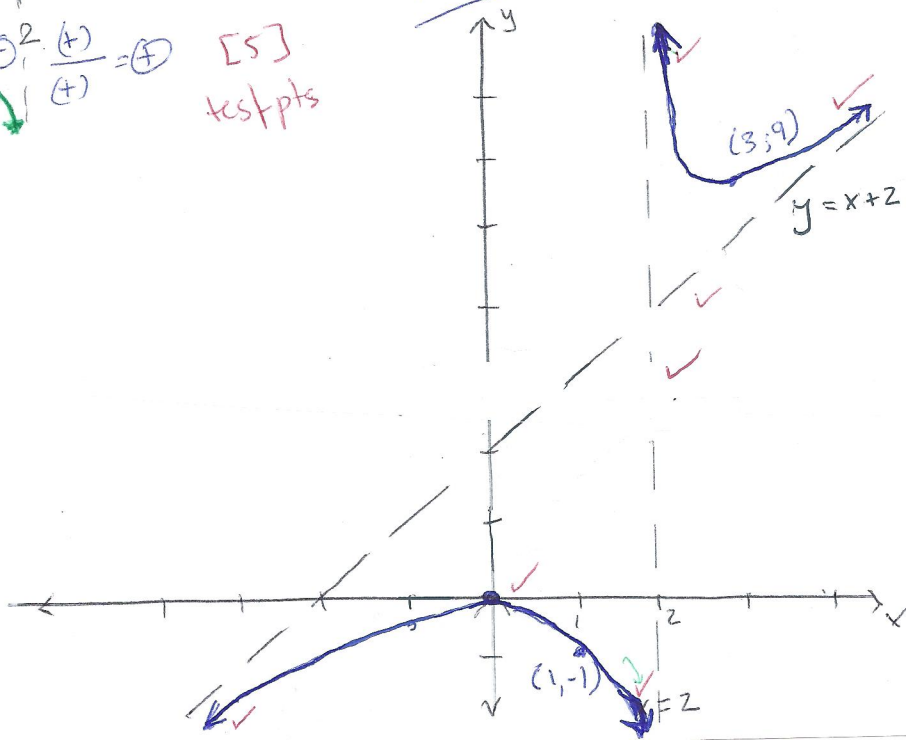
the slant asymptote is $y = x + 2$ [5]

Test Points



[5] testpts

sketch [10]



extra points

x	r(x)
-10	-8.3
-1	-0.33
1	-1
3	9

③ b) $r(x) = \dfrac{x^2-2x-8}{x} = \dfrac{(x+2)(x-4)}{x}$

x-int: $r(x)=0$ when $(x+2)(x-4)=0$

↳ $x=-2, 4$ → (-2, 0), (4, 0)

y-int: $r(0)$ – undefined → no y-int

VA: where denom. $=0$ → V.A @ $x=0$

HA: none b/c deg(N) > deg(D)

↳ long division: (don't really need long division to do that)

$$x \overline{)\, x^2-2x-8} \quad x-2-\frac{8}{x}$$

x^2

$0-2x$

$-2x$

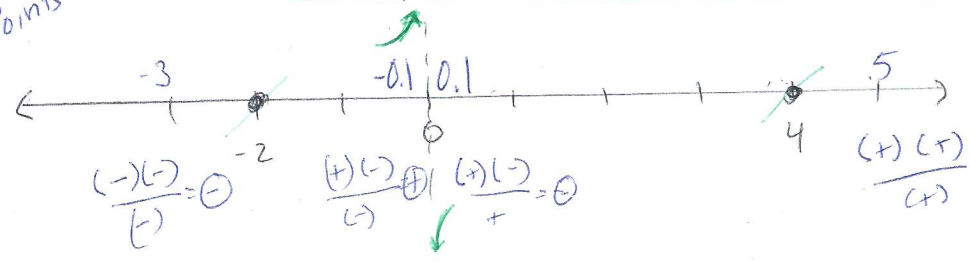
$0-8$

Since $\dfrac{x^2-2x-8}{x} = x-2-\dfrac{8}{x}$ and as $x \to \pm\infty$, $\dfrac{8}{x} \to 0$,

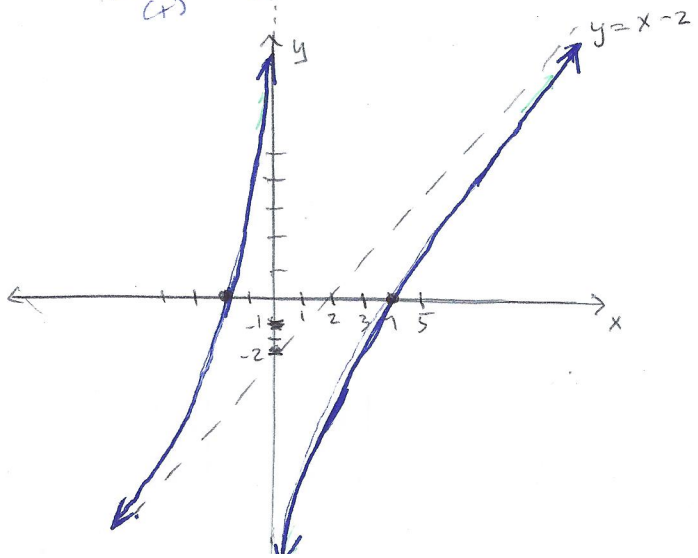
the slant asymptote is $y = x-2$ [5]

[5]

Test Points



sketch [10]



③ c) $r(x) = \dfrac{x^3+x^2}{x^2-4} = \dfrac{x^2(x+1)}{(x-2)(x+2)}$

x-int: $r(x)=0$ when $x^2(x+1)=0$

$x=0$ (multiplicity = 2)

$x=-1$

$(0,0) \leftarrow m=2$ y-int & x-int

$(-1,0)$ x-int.

[5]

y-int: $r(0)=0$

VA: set denominator $=0$ $(x-2)(x+2)=0$

$x=2, -2$ → VA @ $x=2$, $x=-2$

HA: none b/c $\deg(N)=3 > \deg(D)=2$

↳ long division

$$x^2+0x-4 \;\overline{)\; x^3+x^2+0x+0}$$

$x + 1 + \dfrac{4x+4}{x^2-4}$ → slant asymptote is $y = x+1$

$x^3 + 0x^2 - 4x$ ↓

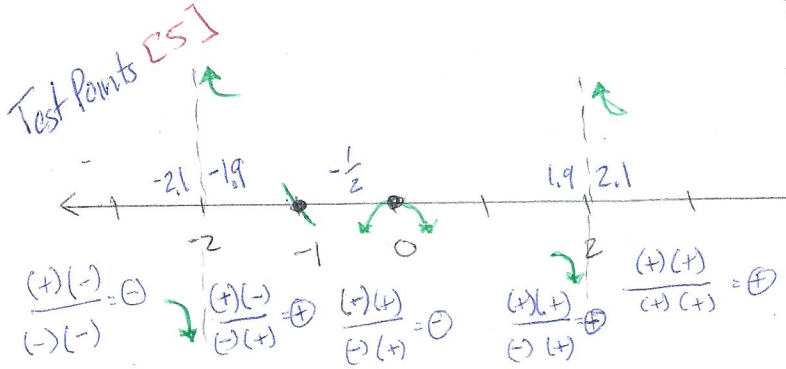
$x^2 + 4x + 0$

$x^2 + 0x - 4$

$4x + 4$

[5]

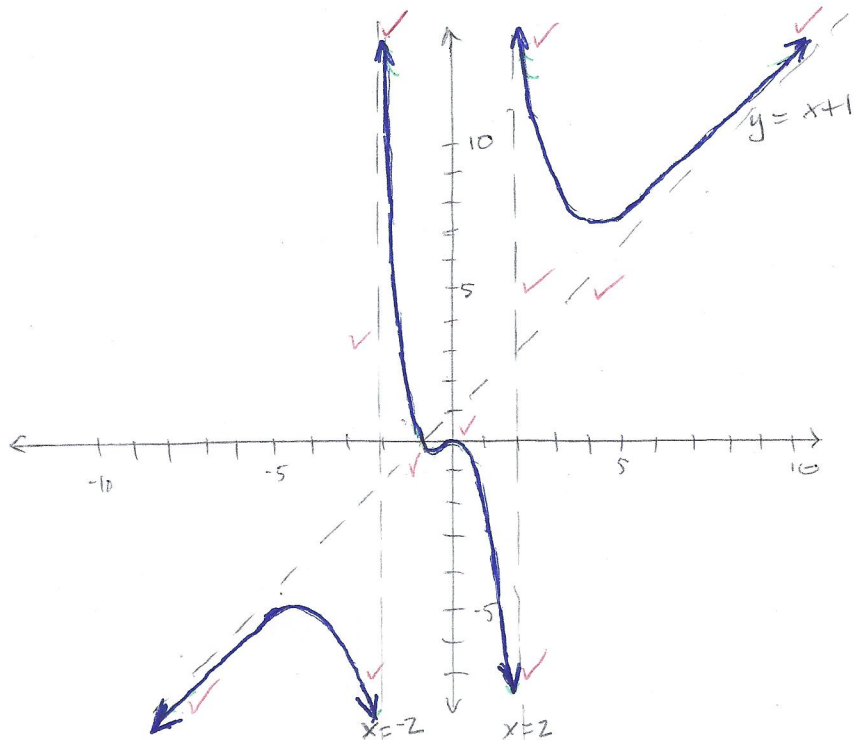
Test Points [5]



extra points

x	
-5	-4.76
-1.5	0.64
-½	-0.03
5	7.14

Sketch [10]



(4) Graph needs - slant asymptote: $y = \frac{1}{2}x - \frac{1}{4}$

(15 pt)

- VA: $x = -1$, $x = \frac{1}{2}$ only
- x-int: $(-\sqrt[3]{4}, 0) \approx (-1.587, 0)$ only
- VA behavior: