Math 113

Group Quiz 14 Solutions

① # of polynomials of degree 5 with zeros, $-2, -1, 0, 1, 2$: $\boxed{\infty}$ [3]

What makes them different? $\boxed{\text{the coefficient in front of the factors}}$

[12]

$$P(x) = 1\,(x+2)(x+1)\,x\,(x-1)(x-2)$$

$$R(x) = -100\,(x+2)(x+1)\,x\,(x-1)(x-2)$$

$$T(x) = 24\,(x+2)(x+1)\,x\,(x-1)(x-2)$$

list 3 polynomials with the given factors and distinct coefficients.

② # of polynomials of degree 5 with zeros: $-1, 0, 1$: $\boxed{\infty}$ [3]

What makes them different?

- The coefficient in front of the factors
- The powers (or multiplicity) of each factor.

[12]

$$P(x) = 1\,(x+1)^3\,x\,(x-1)$$

$$R(x) = -100\,(x+1)\,x^3(x-1)$$

$$T(x) = 24\,(x+1)\,x^2(x-1)^2$$

list 3 polynomials with the given factors, distinct coefficients and/or distinct powers.

③ Want a poly of degree 3 with zeros 1, −6, 4
with coeff. of x-term 5.

Let $P(x) = A(x-1)(x+6)(x-4)$ [5]

$= A(x^2+6x-x-6)(x-4)$

$= A(x^2+5x-6)(x-4)$

↓ just care about the x term

$= A(-20x-6x+\cdots)$

$= A(-26x+\cdots)$

$= \underbrace{-26A}_{\text{needs to be } 5.}x + A(\cdots)$

$-26A = 5$

$A = -\frac{5}{26}$ [5]

$\therefore \boxed{P(x) = -\frac{5}{26}(x-1)(x+6)(x-4)}$ [5]

(4) Want Poly of degree 4, zeros: 1, −1 with $m=2$ each.

& passes through $(2, -18)$

let $P(x) = A(x-1)^2(x+1)^2$ [5]

we know $P(2) = A(2-1)^2(2+1)^2 = -18$

$A(1)(9) = -18$

$A = -2$ [5]

$\therefore \boxed{P(x) = -2(x-1)^2(x+1)^2}$ [5]

(5) Want poly of degree 3, zeros -4, 1

& passes through $(0, -12)$ $\quad m=1 \quad m=2$

because of the graph's behavior at $x=1$ we know the multiplicity here is even. Thus must be 2 since the poly is of degree 3.

let $P(x) = A(x+4)(x-1)^2$ [5]

know $P(0) = A(4)(-1)^2 = -12$

$A = -3$ [5]

$\boxed{P(x) = -3(x+4)(x-1)^2}$ [5]

⑥ want Poly of deg 4 with zeros: -4, 2

$\quad m=2 \quad m=2$

& passes through $(0, -4)$

---

let $P(x) = A(x+4)^2(x-2)^2$ [5] the graph's behavior tells us that the multiplicity at each zero must be 2.

know $P(0) = A(4^2)(-2)^2 = -4$

$64A = -4$

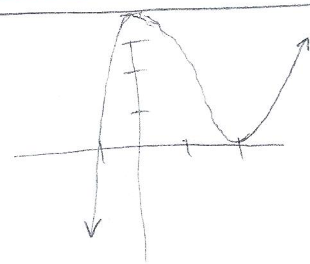
$A = \frac{-4}{64} = \frac{-1}{16}$ [5]

$\boxed{P(x) = \frac{-1}{16}(x+4)^2(x-2)^2}$ [5]

---

⑦ $f(x) = (x+1)(x-2)^2$

(10pt)



why can't $g(x) = -2(x+1)(x-2)^2$ have such a graph?

all acceptable answers

- the end behavior for $g(x)$ is opposite of $f(x)$.
- the behavior near the zeros of $g(x)$ would be opposite.
- $g(x) = -2f(x)$, so the graph would be reflected about the x-axis. and stretched vertically by a factor of 2.
- The coefficients of $f(x)$ & $g(x)$ are different, which necessarily changes the graph.