Math 113

Group Quiz 13 Solutions

① | Leading Term | Looks Like | End Behavior | Zeros

[4] [6]

a) $P(x) = x^3 + 2x^2 - 8x$ x^3

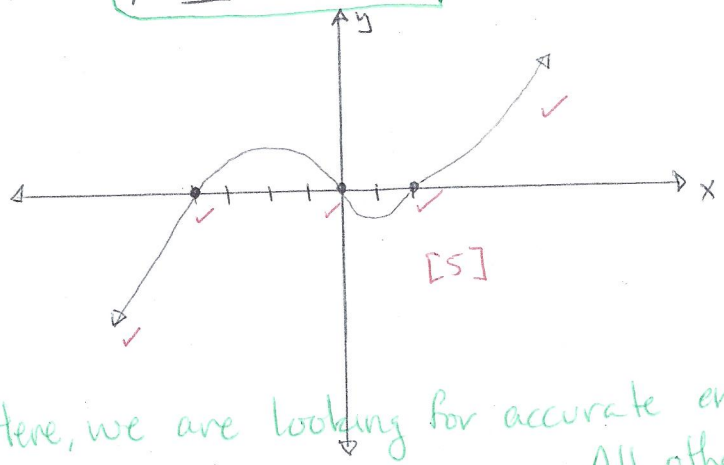
$= x(x^2 + 2x - 8)$

$= x(x+4)(x-2)$ [5]

$y \to \infty$ as $x \to \infty$

$y \to -\infty$ as $x \to -\infty$

$x = -4, 0, 2$



[5]

$P(x) = 0$ when

$x(x+4)(x-2) = 0$

$x = 0$ OR $x = -4$ OR $x = 2$

Here, we are looking for accurate end behavior & accurate zeros. All other details will become more precise in the coming weeks.

b) $P(x) = x^4 - 2x^3 + 8x - 16$ x^4

$= x^3(x-2) + 8(x-2)$

$= (x^3 + 8)(x-2)$

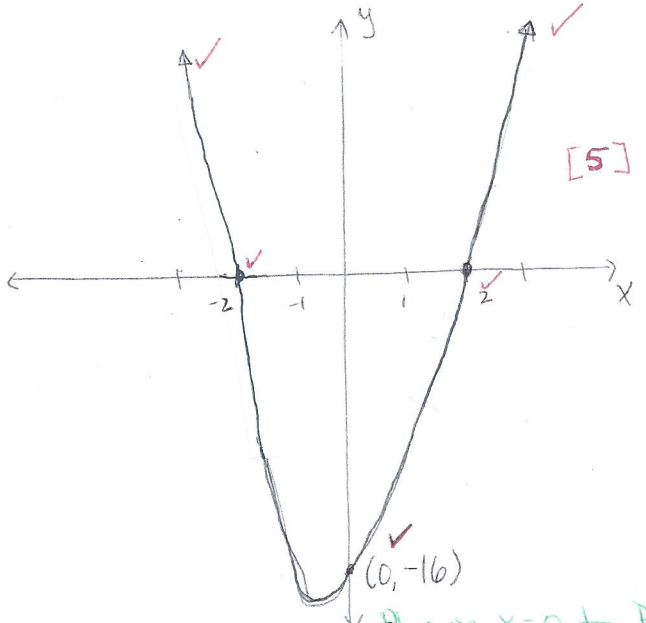
$= (x+2)(x^2 - 2x + 4)(x-2)$ [5]

$y \to \infty$ as $x \to \infty$

$y \to \infty$ as $x \to -\infty$

[4]

$x = -2, 2$ [6]



[5]

Plug in $x = 0$ to $P(x)$ to find this additional point.

$P(x) = 0$ when

$(x+2) = 0$ or $(x^2 - 2x + 4) = 0$ or $(x-2) = 0$

$x = -2$ no solutions $x = 2$

notice that $x^3 + 8$ is the sum of two cubes: $x^3 + 2^3$.

It can be factored using the sum of two cubes rule:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Therefore

$x^3 + 8 = (x+2)(x^2 - 2x + 4)$

(2)

Zeros	multiplicity;
$x = -4$	$m = 1$, (b) odd, =1
$x = -3$	$m = 1$ (b) odd, =1
$x = -1$	$m = 3$, (c) odd, >1
$x = 3$	$m = 2$, (a) even

[2, 1, 1]

[2, 1, 1]

[2, 1, 1]

[2, 1, 1]

The sum of the multiplicities (which is equal to the degree of the polynomial) is 7. [2]

[2]

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(3) $P(x) = -(x-1)^2(x+2)^3$

step 1

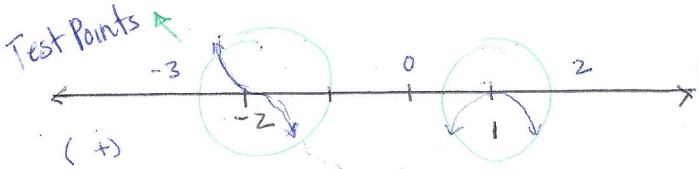
Zeros: $x = 1 \rightarrow m = 2$, $x = -2 \rightarrow m = 3$ [4]

step 4

x	sign	P(x)
-3	+	$-(-4)^2(-1)^3 = 16$
-2		0
0	−	$-(-1)^2(2)^3 = -8$
1		0
2	−	$-(1)^2(4)^3 = -64$

[2]

step 2



[3]

notice the way it meets the zero because of multiplicity.

step 3

End Behavior

leading term: $-x^5$

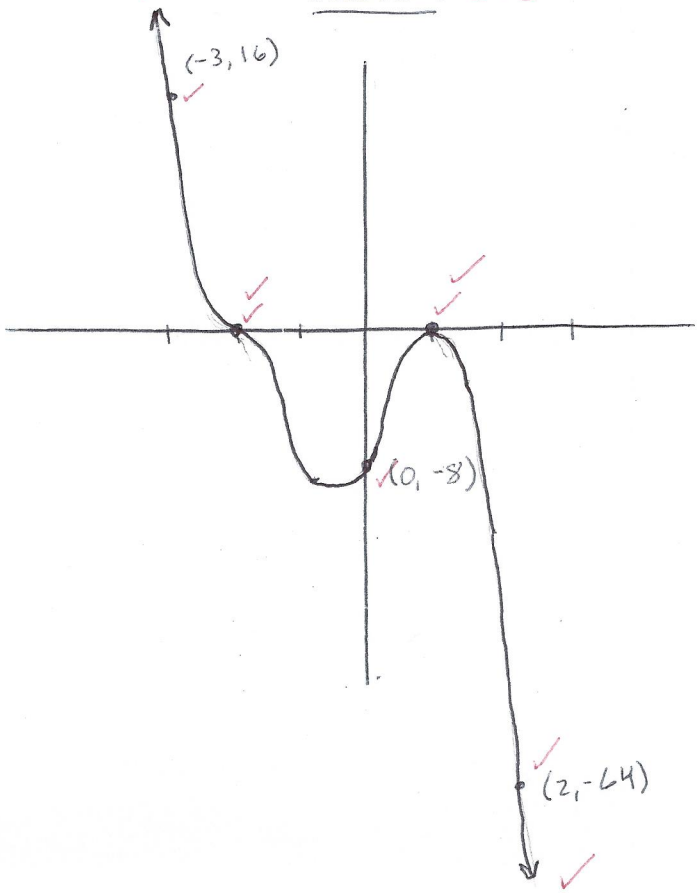
looks like: ↗↘

$y \rightarrow -\infty$ as $x \rightarrow \infty$

$y \rightarrow \infty$ as $x \rightarrow -\infty$

[2]

sketch [9]



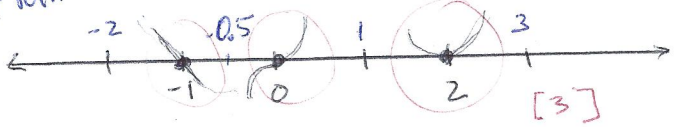
(4) $P(x) = 2x^3(x+1)(x-2)^2$

step 1 zeros:

$x = 0 \rightarrow$	$m = 3$
$x = -1 \rightarrow$	$m = 1$
$x = 2 \rightarrow$	$m = 2$

[3]

step 2 Test Points



[3]

step 4

x	sign	P(x)
-2	+	$2(-2)^3(-1)(-3)^2 = 144$
-1		0
-0.5	-	$2(-0.5)^3(0.5)(-2.5)^2 = -0.78125$
0		0
1	+	$2(1)^3(2)(-1)^2 = 4$
2		0
3	+	$2(3)^3(4)(1)^2 = 216$

step 3 End Behavior

Leading term: $2x^6$

looks like: ∪

$y \rightarrow \infty$ as $x \rightarrow \infty$

$y \rightarrow \infty$ as $x \rightarrow -\infty$

[2]

Sketch [12]