Math 113

Group Quiz 11 Solutions

① [16 pt, 4 each]

a) $\sqrt[7]{13}$ translates to $\boxed{13^{1/7}}$ in exponential form

b) $4^{1/3}$ translates to $\boxed{\sqrt[3]{4}}$ in radical form.

c) $\frac{1}{4^5}$ translates to $\boxed{4^{-5}}$ in exponential form.

d) 7^{-5} is the same as $\boxed{\frac{1}{7^5}}$ in fraction form.

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② [24 pt, 3 each]

a) $x^5 = 32$

$(x^5)^{1/5} = (2^5)^{1/5}$

$x = 2^{5 \cdot \frac{1}{5}}$

$\boxed{x = 2}$

b) $x^6 = 64$

$(x^6)^{1/6} = \pm(2^6)^{1/6}$

notice the moment we take the 6th root, we include the ± options.

$x = \pm 2^{6 \cdot \frac{1}{6}}$

$\boxed{x = \pm 2}$

c) $x^5 = -32$

$(x^5)^{1/5} = ((-2)^5)^{1/5}$

$\boxed{x = -2}$

d) $x^6 = -64$

$\boxed{\text{No Solutions}}$

We know that any number raised to the 6th power will be positive.

e) $(x^{1/2})^2 = (4)^2$

$\boxed{x = 16}$

f) $x^{1/2} = -4$

$\boxed{\text{no solution}}$

we know the square root of any positive number yields the positive solution.

g) $(x^{1/3})^3 = (4)^3$

$\boxed{x = 64}$

h) $(x^{1/3})^3 = (-4)^3$

$\boxed{x = -64}$

(3) a) $x^8 - 8x^4 + 7 = 0$

Let $w = x^4 \rightarrow w^2 = (x^4)^2 = x^8$ start with a change of variable. [2]

$w^2 - 8w + 7 = 0$

$1 \quad -1 \quad -1$
$1 \quad 7 \quad -7$

$(w-1)(w-7) = 0$ [2]

$w - 1 = 0$ or $w - 7 = 0$

$w = 1$ $\quad$ $w = 7$ remember to change back to the original variable.

$(x^4)^{1/4} = \pm(1)^{1/4}$ $\quad$ $(x^4)^{1/4} = \pm(7)^{1/4}$

$\boxed{x = \pm 1 \text{ or } x = \pm\sqrt[4]{7}}$ [4]

no fractions in original, so no extraneous solutions ✓

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b) $4x^6 - 12x^3 + 9 = 0$

Let $w = x^3 \rightarrow w^2 = (x^3)^2 = x^6$ change variable [2]

$4w^2 - 12w + 9 = 0$

$1 \; \boxed{2} \qquad 1 \; \boxed{-3} \; -6$
$4 \; \boxed{2} \qquad 9 \; \boxed{-3} \; -6$

$(2w-3)(2w-3) = 0$ [2]

$2w - 3 = 0$

$2w = 3$

$w = \frac{3}{2}$

$(x^3)^{1/3} = \left(\frac{3}{2}\right)^{1/3}$ switch back to original variable

[4]

$\boxed{x = \sqrt[3]{\frac{3}{2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2 \cdot 2}}{\sqrt[3]{2 \cdot 2}} = \frac{\sqrt[3]{12}}{2}}$ rationalize the denominator.

no extraneous solutions ✓

3) c) $x^{4/3} - 5x^{2/3} + 6 = 0$

let $w = x^{2/3} \Rightarrow w^2 = (x^{2/3})^2 = x^{4/3}$ [2]

$w^2 - 5w + 6 = 0$

$(w - 2)(w - 3) = 0$ [2]

$w = 2$ $\qquad$ $w = 3$

$(x^{2/3})^{3/2} = (2)^{3/2}$ $\qquad$ $(x^{2/3})^{3/2} = (3)^{3/2}$

$\boxed{x = \sqrt{8} \text{ or } x = \sqrt{27}}$ [4]

no extraneous solutions ✓

d) $\left(\frac{x+1}{x}\right)^2 + 4\left(\frac{x+1}{x}\right) + 3 = 0$

let $w = \left(\frac{x+1}{x}\right) \Rightarrow w^2 = \left(\frac{x+1}{x}\right)^2$ [2]

$w^2 + 4w + 3 = 0$

$(w + 3)(w + 1) = 0$ [2]

$w + 3 = 0$ $\qquad$ $w + 1 = 0$

$w = -3$ $\qquad$ $w = -1$

$\frac{x+1}{x} = -3$ $\qquad$ $\frac{x+1}{x} = -1$

$x + 1 = -3x$ $\qquad$ $x + 1 = -x$

$4x = -1$ $\qquad$ $2x = -1$

$\boxed{x = -\frac{1}{4} \text{ or } x = -\frac{1}{2}}$ [4]

no extraneous solutions ✓

3]

③ e) $2x^3 + x^2 - 18x - 9 = 0$ Factor by Grouping!

$\underline{x^2(2x+1)} - \underline{9(2x+1)} = 0$ [2]

$(x^2-9)(2x+1) = 0$

$(x-3)(x+3)(2x+1) = 0$ [3]

$\downarrow \quad \downarrow \quad \searrow$

$x = 3 \quad x = -3 \quad 2x = -1$

$x = -\frac{1}{2}$ [3]

no extraneous solutions ✓

f) $x^3 - 5x^2 + 6x = 0$ Start by factoring out the common x factor.

$x(x^2 - 5x + 6) = 0$ [2]

$x(x-2)(x-3) = 0$ [3]

$\downarrow \quad \searrow \quad \searrow$

$\boxed{x = 0 \quad x = 2 \quad x = 3}$ [3]

(4) a) 2 real solutions:

4 each Correct process + a correct answer

pick any 2 solutions: $x = 1 \ , \ x = -2$

Work backwards: $x - 1 = 0 \qquad x + 2 = 0$

Use the zero product rule $(x-1)(x+2) = 0$

distribute until you $x^2 + 2x - x - 2 = 0$

are in quadratic form $x^2 + x - 2 = 0$

b) exactly one solution:

pick your one solution: $x = 1$

Use the solution twice $x - 1 = 0$

$$(x-1)(x-1) = 0$$

$$x^2 - x - x + 1 = 0$$

$$x^2 - 2x + 1 = 0$$

c) No real solution:

recall: we get no real solution when the Discriminant is less than 0.

$$D = b^2 - 4ac < 0$$

So pick any a, b, & c which would satisfy this.

For example: let $b = 0$, $a = 1$, $c = 1$

$$\rightarrow D = 0^2 - 4 \cdot 1 \cdot 1 = -4 < 0$$

$$\hookrightarrow 1x^2 + 0x + 1 = 0$$

5

$\boxed{x^2 + 1 = 0}$ has no real solutions