Math 113

Group Quiz 11 Solutions

1. a) \( \sqrt[3]{13} \) translates to \( 13^{\frac{1}{3}} \) in exponential form.

b) \( 4^{\frac{1}{3}} \) translates to \( \sqrt[3]{4} \) in radical form.

c) \( \frac{1}{4^5} \) translates to \( 4^{-5} \) in exponential form.

d) \( 7^{-5} \) is the same as \( \frac{1}{7^5} \) in fraction form.

2. a) \( x^5 = 32 \)
   \[ (x^5)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} \]
   \[ x = 2^{5 \cdot \frac{1}{5}} \]
   \[ x = 2 \]

b) \( x^6 = 64 \)
   \[ (x^6)^{\frac{1}{6}} = (2^6)^{\frac{1}{6}} \]
   \[ x = \pm 2^{6 \cdot \frac{1}{6}} \]
   \[ x = \pm 2 \]

c) \( x^5 = -32 \)
   \[ (x^5)^{\frac{1}{5}} = (-2^5)^{\frac{1}{5}} \]
   \[ x = -2 \]

d) \( x^6 = -64 \)
   \[ \text{No Solutions} \]
   
   We know that any number raised to the 6th power will be positive.

e) \( (x^{\frac{1}{2}})^2 = (4)^2 \)
   \[ x = 16 \]

f) \( x^{\frac{1}{2}} = -4 \)
   \[ \text{No Solution} \]
   
   We know the square root of any positive number yields the positive solution.

g) \( (x^{\frac{1}{3}})^3 = (4)^3 \)
   \[ x = 64 \]

h) \( (x^{\frac{1}{3}})^3 = (-4)^3 \)
   \[ x = -64 \]
\(3\)  
\(a) \quad x^8 - 8x^4 + 7 = 0\)

Let \(w = x^4 \rightarrow w^2 = (x^4)^2 = x^8\)

\[w^2 - 8w + 7 = 0\]

\[\begin{align*}
1 & -1 -1 \\
1 & -7 -7
\end{align*}\]

\[(w - 1)(w - 7) = 0\]

\(w - 1 = 0 \quad \text{or} \quad w - 7 = 0\)

\(w = 1 \quad \text{or} \quad w = 7\)

\((x^4)^{\frac{1}{4}} = \pm \left(7\right)^{\frac{1}{4}}\)

\(x = \pm 1 \quad \text{or} \quad x = \pm \sqrt[4]{7}\)

No fractions in original, so no extraneous solutions.

Remember to change back to the original variable.

\(b) \quad 4x^6 - 12x^3 + 9 = 0\)

Let \(w = x^3 \rightarrow w^2 = (x^3)^2 = x^6\)

\(4w^2 - 12w + 9 = 0\)

\[\begin{align*}
\underline{12} & \\
\underline{4} & 3
\end{align*}\]

\[(2w - 3)(2w - 3) = 0\]

\(2w - 3 = 0\)

\(2w = 3\)

\(w = \frac{3}{2}\)

\((x^3)^{\frac{1}{3}} =\left(\frac{3}{2}\right)^{\frac{1}{3}}\)

\(x = \sqrt[3]{\frac{3}{2}} = \frac{3^{\frac{1}{3}}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{\sqrt[3]{12}}{2}\)

No extraneous solutions.
c) \( x^{\frac{4}{3}} - 5x^{\frac{2}{3}} + 6 = 0 \)

Let \( w = x^{\frac{2}{3}} \) \( \Rightarrow w^2 = (x^{\frac{2}{3}})^2 = x^{\frac{4}{3}} \)  [2]

\( w^2 - 5w + 6 = 0 \)

\((w - 2)(w - 3) = 0\)  [2]

\( w = 2 \quad w = 3 \)

\( (x^{\frac{2}{3}})^{\frac{3}{2}} = (2)^{\frac{3}{2}} \quad (x^{\frac{2}{3}})^{\frac{3}{2}} = (3)^{\frac{3}{2}} \)

\[ x = \sqrt{8} \quad \text{or} \quad x = \sqrt{27} \]  [4]

No extraneous solutions √

d) \( \left( \frac{x+1}{x} \right)^2 + 4\left( \frac{x+1}{x} \right) + 3 = 0 \)

Let \( w = \left( \frac{x+1}{x} \right) \) \( \Rightarrow w^2 = \left( \frac{x+1}{x} \right)^2 \)  [2]

\( w^2 + 4w + 3 = 0 \)

\((w + 3)(w + 1) = 0\)  [2]

\( w + 3 = 0 \quad w + 1 = 0 \)

\( w = -3 \quad w = -1 \)

\( \frac{x+1}{x} = -3 \quad \frac{x+1}{x} = -1 \)

\( x+1 = -3x \quad x+1 = -x \)

\( 4x = -1 \quad 2x = -1 \)

\[ x = -\frac{1}{4} \quad \text{or} \quad x = -\frac{1}{2} \]  [4]

No extraneous solutions √
(3) \[ e) \quad 2x^3 + x^2 - 18x - 9 = 0 \]

Factor by Grouping!

\[ x^2(2x+1) - 9(2x+1) = 0 \]

\[ (x^2 - 9)(2x+1) = 0 \]

\[ (x-3)(x+3)(2x+1) = 0 \]

\[ x = 3 \quad x = -3 \quad 2x = -1 \quad x = -\frac{1}{2} \]

no extraneous solutions

(3)

\( 2x^3 + x^2 - 18x - 9 = 0 \)

\[ x(2x^2 - 5x + 6) = 0 \]

\[ x(x - 2)(x - 3) = 0 \]

\[ x = 0 \quad x = 2 \quad x = 3 \]
a) 2 real solutions:
   pick any 2 solutions: $x = 1, x = -2$
   Work backwards: $x - 1 = 0$ \[\Rightarrow\] $x + 2 = 0$
   Use the zero product rule
   distribute until you are in quadratic form
   
   $x^2 + 2x - x - 2 = 0$
   $x^2 + x - 2 = 0$

b) exactly one solution:
   pick your one solution: $x = 1$
   Use the solution twice
   
   $x - 1 = 0$
   \[\Rightarrow\]
   $(x - 1)(x - 1) = 0$
   $x^2 - x - x + 1 = 0$
   $x^2 - 2x + 1 = 0$

c) No real solution:
   recall: we get no real solution when the discriminant is less than 0.
   
   $D = b^2 - 4ac < 0$

   So pick any $a, b, c$ which would satisfy this.
   For example. let $b = 0$, $a = 1$, $c = 1$
   \[\Rightarrow\] $D = 0^2 - 4.1.1 = -4 < 0$

   \[\Rightarrow\] $x^2 + 0x + 1 = 0$
   $x^2 + 1 = 0$ has no real solutions